

Charged particle in a magnetic field

1 Introduction

In the following, the path of a charged particle flying through a magnetic field is simulated by an analog computer.¹ Basically, a particle with charge q moving in a magnetic field $\vec{B}$ is subjected to a force $\vec{F}_{\text{Lorentz}}$ which is perpendicular to the magnetic field $\vec{B}$ and the direction in which the particle moves. This yields

$$\vec{F}_{\text{Lorentz}} = q \left(\vec{v} \times \vec{B} \right) = q \left(\dot{\vec{r}} \times \vec{B} \right),$$

where $\vec{v}$ denotes the velocity and $\vec{r}$ the position of the particle under consideration. On the other hand

$$\vec{F}_{\text{particle}} = m\ddot{\vec{r}}.$$

In addition to this, some form of friction exerting a force

$$\vec{F}_{\text{friction}} = -\mu\dot{\vec{r}}$$

is assumed to act on the particle.

These three equations finally yield

$$m\ddot{\vec{r}} = \vec{F}_{\text{Lorentz}} + \vec{F}_{\text{friction}} = q \left(\dot{\vec{r}} \times \vec{B} \right) - \mu\dot{\vec{r}}. \quad (1)$$

It is further assumed that the particle moves in the x - y plane to which the magnetic field is perpendicular. With e_x , e_y , and e_z denoting the

¹This application note is based on an old application note by Telefunken, cf. [Telefunken/Particle], which was quite probably written by Ms. INGE BORCHARDT, who did the trajectory simulations at DESY for the high-energy particle accelerators on an EAI 231RV analog computer, and later on a Telefunken RA770 hybrid computer.

unit vectors pointing into x -, y -, and z -direction, this yields

$$\vec{B} = B_z e_z \text{ and}$$

$$\dot{\vec{r}} = \dot{x}e_x + \dot{y}e_y$$

respectively. Accordingly, it is

$$\dot{\vec{r}} \times \vec{B} = \dot{y}B_z e_x - \dot{x}B_z e_y.$$

Combining this with equation (1) finally yields

$$m\ddot{\vec{r}} = m(\ddot{x}e_x + \ddot{y}e_y) = q(\dot{y}B_z e_x - \dot{x}B_z e_y) - \mu(\dot{x}e_x + \dot{y}e_y). \quad (2)$$

Splitting (2) into components results in

$$m\ddot{x} = q\dot{y}B_z - \mu\dot{x} \text{ and}$$

$$m\ddot{y} = -q\dot{x}B_z - \mu\dot{y}.$$

To simplify things a bit further, two new variables are introduced:

$$\alpha = \frac{q}{m}B_z \text{ and}$$

$$\beta = \frac{\mu}{m},$$

This finally yields

$$\ddot{x} = \alpha\dot{y} - \beta\dot{x} \text{ and} \quad (3)$$

$$\ddot{y} = -\alpha\dot{x} - \beta\dot{y} \quad (4)$$

on which the analog computer setup is based.

2 Programming

Deriving the analog computer program from (3) and (4) is straightforward and yields the setup shown in figure 1.

The two comparators in the middle of the schematic control the area to which the magnetic field is confined. x_1 and x_2 determine the left and right coordinate enclosing the field (setting both to maximum yields a field extending from $-1 \leq x \leq +1$). The x - and y -components of the initial velocity of the particle can be controlled by means of the two potentiometers labelled $\dot{x}_0$ and $\dot{y}_0$. The outputs x and y are connected to an oscilloscope set to x/y -mode.

3 Results

All results shown in the following were obtained with the analog computer set to repetitive operation with an OP-time of 10 ms and time-scale factors $k_0 = 10^3$ set on all four integrators.

Figure 2 shows two typical paths of a particle which is deflected by the magnetic field but not captured. The values for α and β were the same for both pictures. Only x_2 , the coordinate at which the magnetic field ceases to act on the right side, has been changed. The particle in the left picture is fast enough to escape the narrow magnetic field while the wider field in the right picture causes the particle to be reflected.

In figure 3 the path of a particle with heavy friction is shown. The friction is so large that the particle cannot escape the area in which the magnetic field acts.

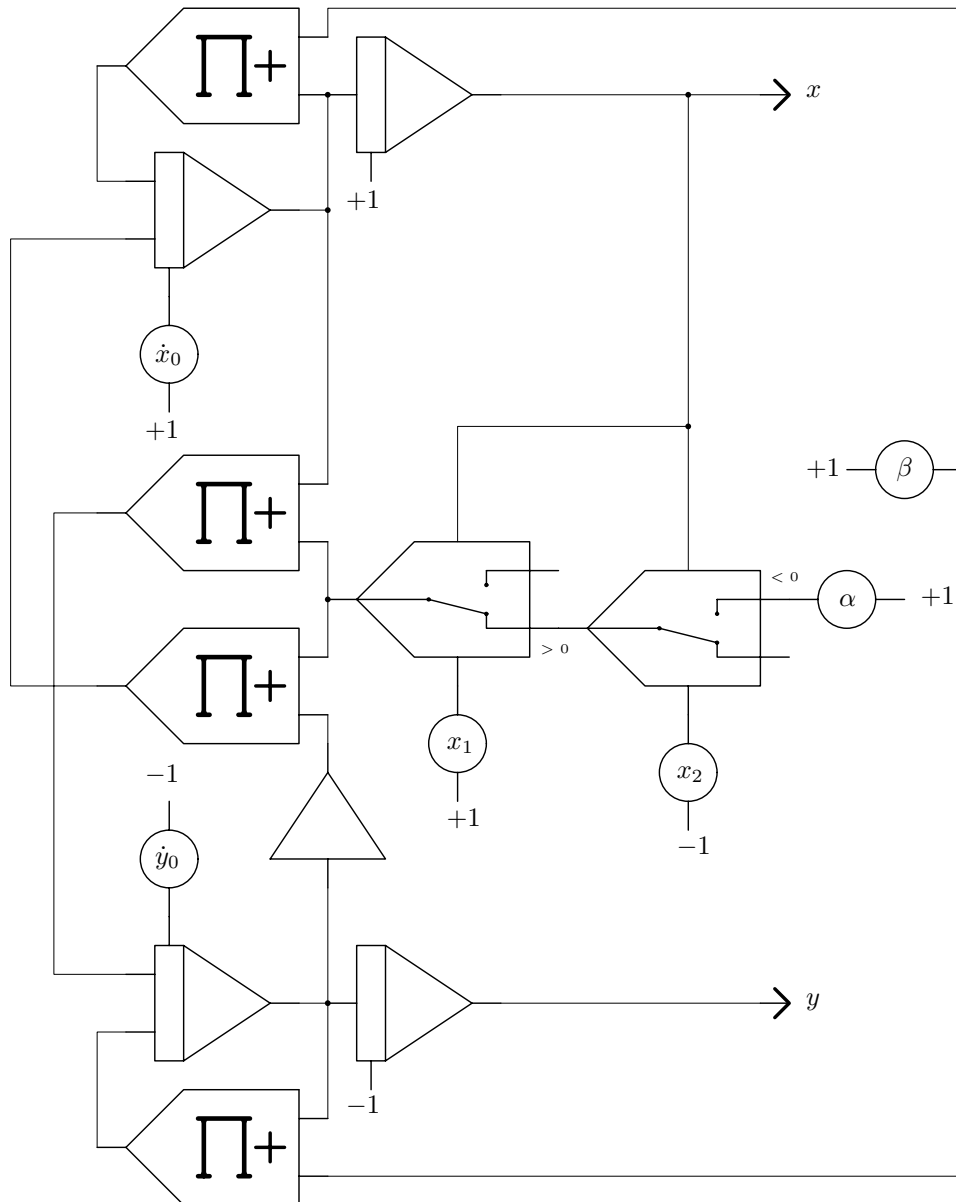


Figure 1: Analog computer setup to simulate the path of a charged particle in a magnetic field

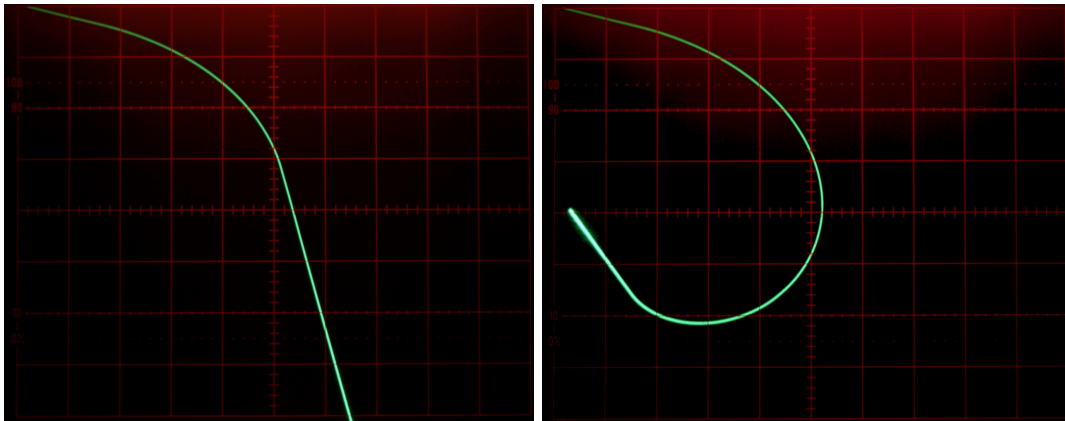


Figure 2: Paths of a particle not captured within the magnetic field

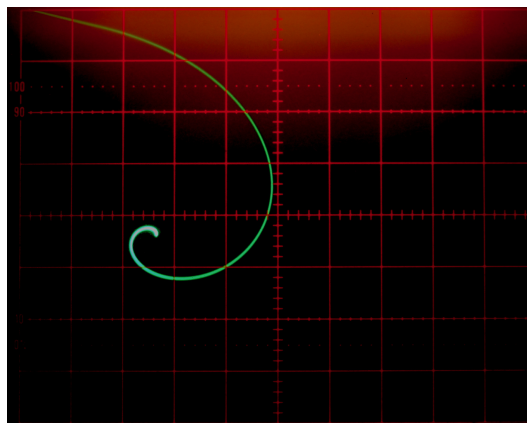


Figure 3: Particle captured by the magnetic field



Analog Computer Applications

References

[Telefunken/Particle] Telefunken, *Demonstrationsbeispiel: Elektrisch geladenes Teilchen im Magnetfeld*, AEG-Telefunken, ADB 007