

## The AIZAWA attractor

This application note implements one of the most beautiful chaotic attractors, the *AIZAWA attractor*.<sup>1</sup> The underlying system consists of three coupled differential equations,

$$\begin{aligned}\dot{x} &= x(z - \beta) - \delta y, \\ \dot{y} &= \delta x + y(z - \beta) \text{ and} \\ \dot{z} &= \gamma + \alpha z - \frac{z^3}{3} + \varepsilon z x^3\end{aligned}$$

with the parameters  $\alpha = 0.095$ ,  $\beta = 0.7$ ,  $\gamma = 0.65$ ,<sup>2</sup>  $\delta = 3.5$ , and  $\varepsilon = 0.1$ . A thorough numerical study of the behavior of this particular system can be found in [LANGFORD 1984].

Scaling these equations to ensure that no variable exceeds the machine units of  $\pm 1$  is pretty straightforward: The scaling factors corresponding to  $x$ ,  $y$ , and  $z$  are  $\lambda_x = \frac{1}{3}$ ,  $\lambda_y = \frac{1}{4}$ , and  $\lambda_z = \frac{1}{2}$ . The value  $z - \beta$  is scaled by  $\frac{2}{5}$ .

The resulting computer setup is a bit convoluted as shown in figure 1. All in all, three integrators, four summers, seven multipliers, and eleven coefficient potentiometers are required for this program.

Figure 2 shows the  $x, z$  phase space plot of the AIZAWA attractor – an exceptionally beautiful structure. To achieve this, two of the parameters, resulting from the scaling, marked by \* and \*\* in figure 1, were varied a bit to achieve a “nicer” picture. The correct values for these are 0.95 and 0.27 respectively.

---

<sup>1</sup>See [COPE 2017].

<sup>2</sup>The original system has  $\gamma = 0.6$  but 0.65 was determined as a better value.

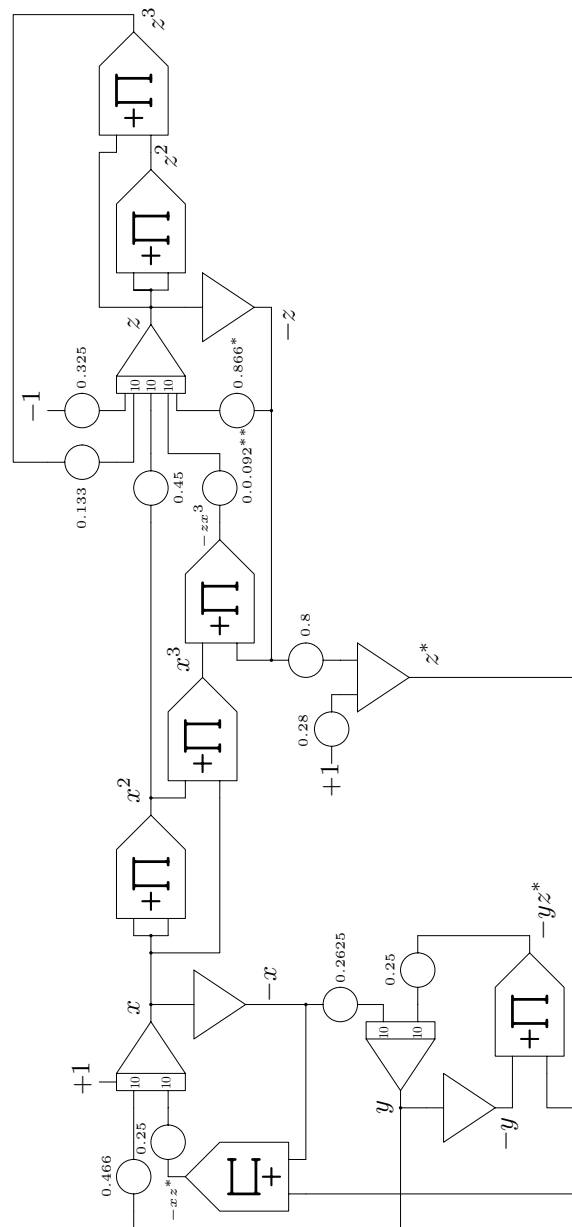


Figure 1: Program for the AIZAWA attractor

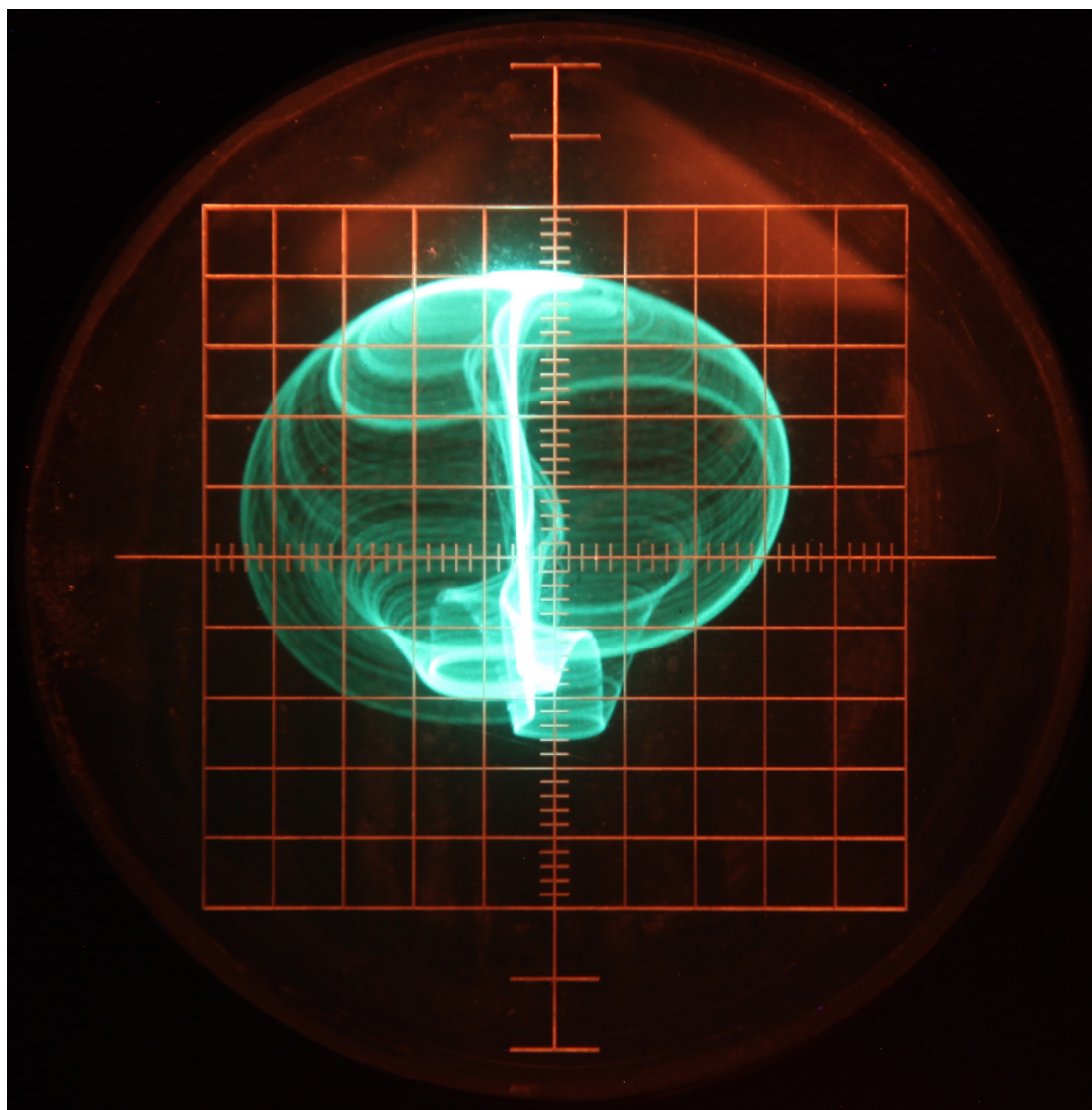


Figure 2: The  $x, z$  phase space plot of the AIZAWA attractor



# Analog Computer Applications

---

## References

- [COPE 2017] GREG COPE, *The Aizawa Attractor*, <http://www.algosome.com/articles/aizawa-attractor-chaos.html>, retrieved 15.12.2018
- [LANGFORD 1984] WILLIAM FINLAY LANGFORD, "Numerical Studies of Torus Bifurcations", in INTERNATIONAL SERIES OF NUMERICAL MATHEMATICS, Vol. 70, 1984 Birkhäuser Verlag Basel, pp. 285–295