

A four-wing attractor

This application note describes a rather beautiful “four-wing attractor”, which is due to [WANG et al. 2009]. It is based on three coupled differential equations of the form

$$\begin{aligned}\dot{x} &= ax + cyz, \\ \dot{y} &= bx + dy - xz, \text{ and} \\ \dot{z} &= ez + fxy.\end{aligned}$$

A particularly beautiful attractor results from the parameter set $a = 0.2$, $b = -0.01$, $c = 1$, $d = -0.4$, $e = -1$, and $f = -1$. The resulting set of equations

$$\begin{aligned}\dot{x} &= 0.2x + yz \\ \dot{y} &= -0.01x - 0.4y - xz \\ \dot{z} &= -z - xy\end{aligned}$$

is easy to scale since a quick numeric experiment shows that x , y , and z are well within the interval $[-3, 3]$, so that scaling coefficients $\lambda_x = \lambda_y = \lambda_z = \frac{1}{3}$ may be used. The resulting scaled system of equations is then

$$\begin{aligned}\dot{x} &= 0.2x + 3yz \\ \dot{y} &= -0.01x - 0.4y - 3xz \\ \dot{z} &= -z - 3xy,\end{aligned}$$

which can be implemented directly on an analog computer.¹

¹Since it requires three products either two coupled THE ANALOG THINGS or an external multiplier using an AD633 or the like are required.



Analog Computer Applications

The resulting analog computer setup is shown in figure 1. Figures 2 and 3 show the behavior of this system in the y/z - and x/y -plane respectively.²

Happy analog computing! :-)

References

[WANG et al. 2009] ZENGHUI WANG, YANXIA SUN, JACOBUS VAN WYK, GUOYUAN QI, MICHAEL ANTONIE VAN WYK, "A 3-D four-wing attractor and its analysis", in *Brazilian Journal of Physics*, vol. 39, no. 3, September, pp. 547–553

²It may be necessary to twiddle the parameter 0.01 slightly to get a nice result.

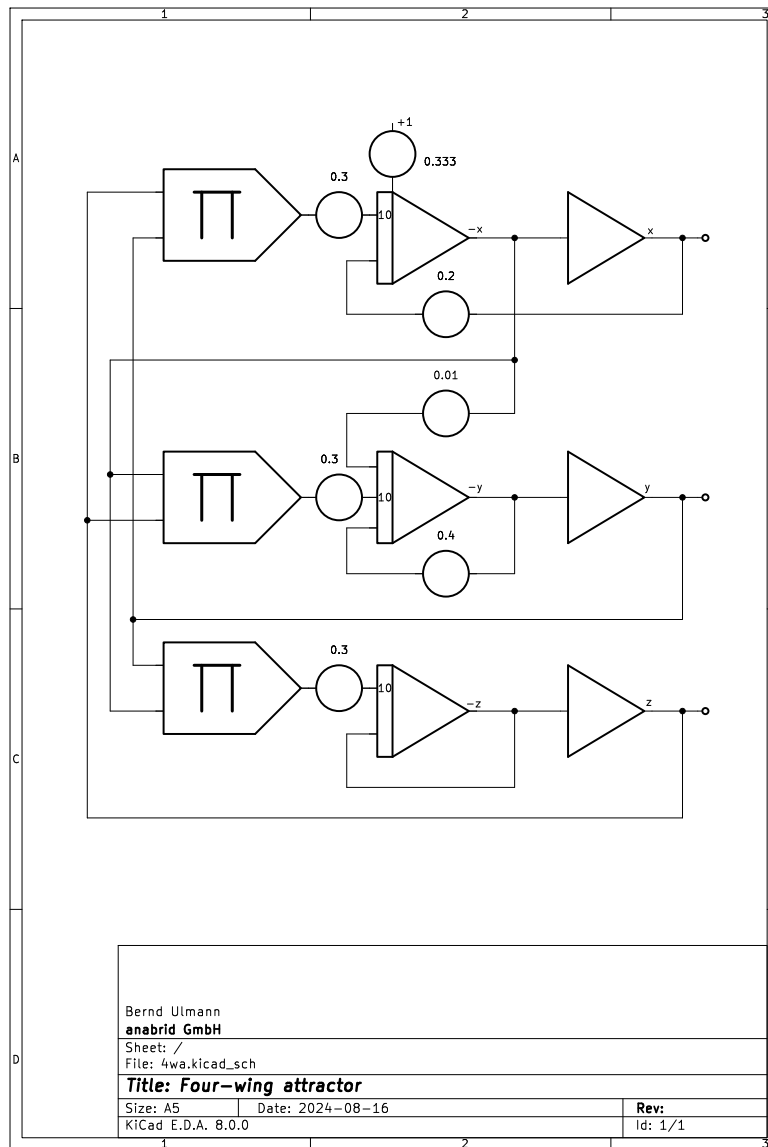


Figure 1: Analog computer setup for the three time scale system

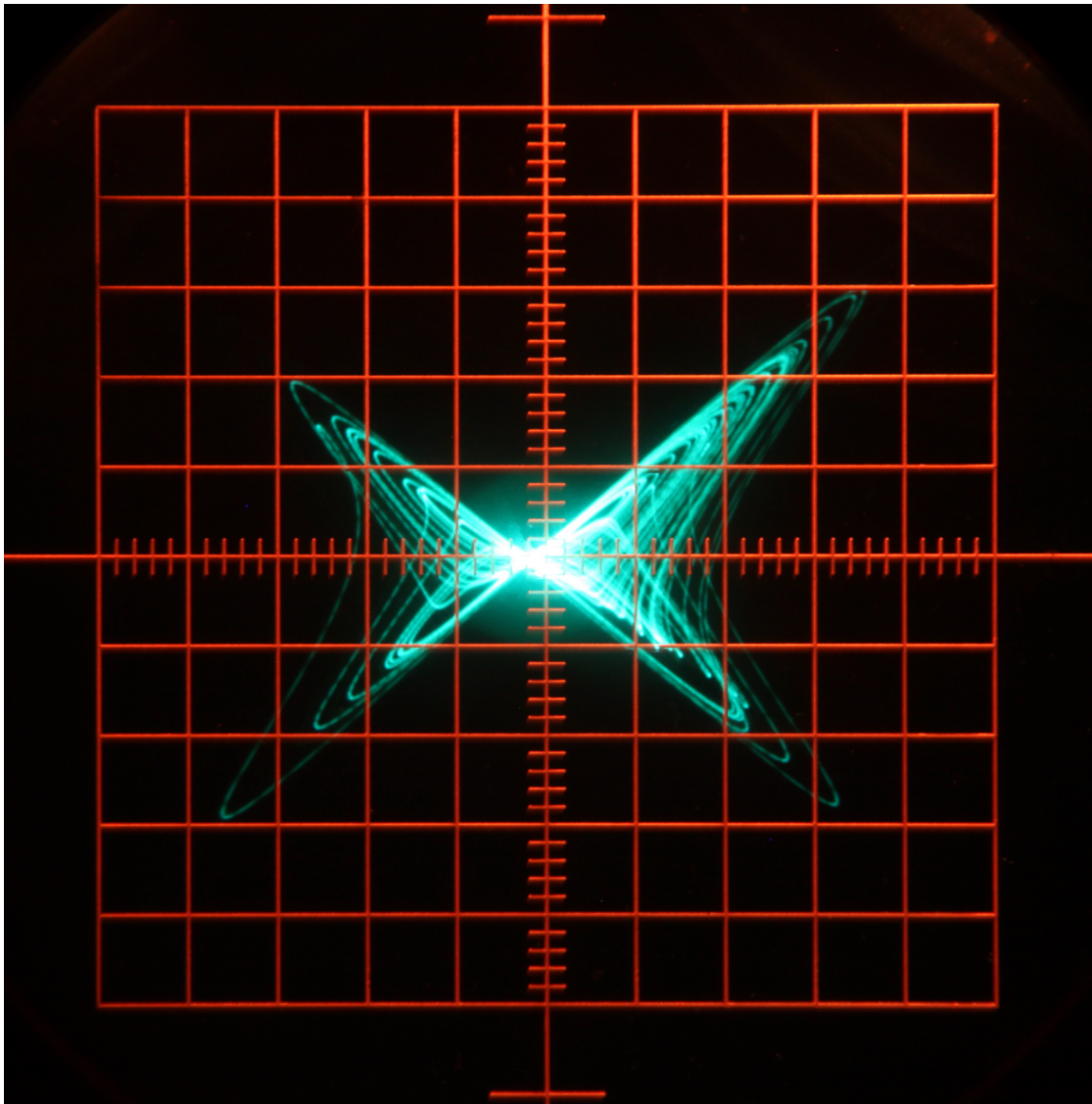


Figure 2: Four-wing attractor in the y/z -plane

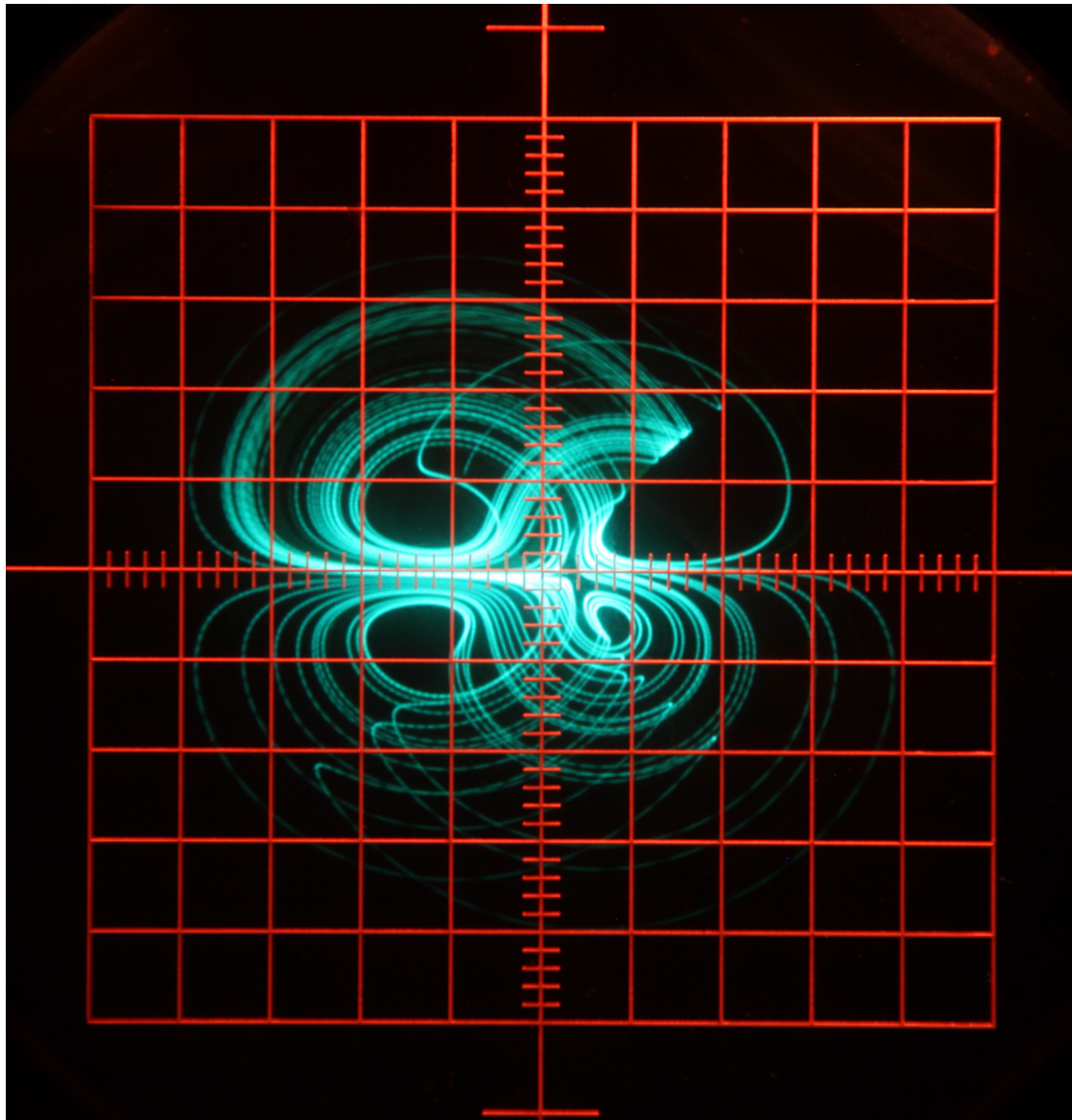


Figure 3: Four-wing attractor in the x/y -plane