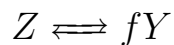
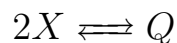
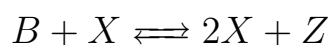
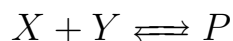
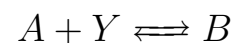




Inspired by the *Oregonator*

1 The problem

The *Oregonator* is a mathematical model for a certain class of autocatalytic chemical reactions. The name is a portmanteau of *Oregon* and *oscillator* as the system was developed by RICHARD J. FIELD and RICHARD M. NOYES¹ at the University of Oregon.² The underlying chemical reactions are:



This was simplified by [FIELD et al. 1974] where the following set of three coupled differential equations can be found:

$$\dot{x} = k_1ay - k_2xy + k_3ax - 2k_4x^2$$

$$\dot{y} = k_1ay - k_2xy + fk_5z$$

$$\dot{z} = k_3ax - k_5z$$

¹See [FIELD et al. 1974].

²Quite like the *Brusselator*, developed in Brussels, see application note #40, https://analogparadigm.com/downloads/alpaca_40.pdf.



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This system is then further simplified further,³ yielding

$$\begin{aligned}\dot{x} &= s(y - xy + y - qx^2), \\ \dot{y} &= \frac{1}{s}(fz - y - xy), \text{ and} \\ \dot{z} &= w(x - z)\end{aligned}$$

with parameters $s = 80$, $q = 10^{-5}$, $w = 0.1$, and $f = 1$.

This already looks nasty (read “stiff”), which is confirmed by a quick numerical experiment showing $\max(x) \approx 10^5$, $\max(y) \approx 2500$, and $\max(z) \approx 3 \cdot 10^4$. This suggests the following scaling factors: $\lambda_x = \lambda_z = 10^{-5}$ and $\lambda_y = 10^{-4}$ resulting in the following system of roughly scaled equations:

$$\begin{aligned}\dot{x} &= \frac{1}{1000}y - 100xy + \frac{1}{100}x - \frac{1}{100}x^2 & k_{0_x} &= 10^3 \\ \dot{y} &= \frac{1}{80}z - \frac{1}{800}y - 125xy & k_{0_y} &= 10 \\ \dot{z} &= \frac{1}{100}x - \frac{1}{100}z & k_{0_z} &= 10\end{aligned}$$

The stiffness shows clearly in the wide spread of coefficients, which are not really practical. This gave rise to some manual experimentation in search of a similar but simpler system, which should also exhibit steep pulses as the Oregonator. The resulting system looks like this and only

³Details are not of interest in the following and can be found in [FIELD et al. 1974].



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has one really large factor of 100 to be taken care of:

$$\dot{x} = 0.01y - 10xy + 0.03x - 0.03x^2 \quad k_{0_x} = 10^3 \quad (1)$$

$$\dot{y} = 0.3z - 100xy \quad k_{0_x} = 10^3 \quad (2)$$

$$\dot{z} = x - z \quad k_{0_z} = 10 \quad (3)$$

2 Implementation

The implementation of this system on THE ANALOG THING is straightforward as shown in figure 1. The factor 100 at the y -integrator is implemented by a 10k resistor connected to the summing junction (SJ) of the corresponding integrator on THE ANALOG THING.⁴

The behavior of the system can be either observed directly with an oscilloscope connected to x , y , and possibly z or using the Arduino hybrid computer library THAThc.⁵ Figure 2 shows an oscilloscope screenshot.⁶

Using the hybrid computer library it is possible to control THE ANALOG THING from an attached Arduino Mega 2560 and gather data during a run. The corresponding Arduino project is shown in figure 3.

With this program loaded into the controlling Arduino, a single run of the analog computer program with data gathering can be performed by issuing the following commands:

```
optime=1500  
ictime=100  
channels=3
```

⁴Inputs with weight 1 have a 1M resistor in THE ANALOG THING.

⁵See <https://github.com/anabrid/THAThc>.

⁶Due to the rather low repetition frequency of several 100 ms a digital storage oscilloscope was used.

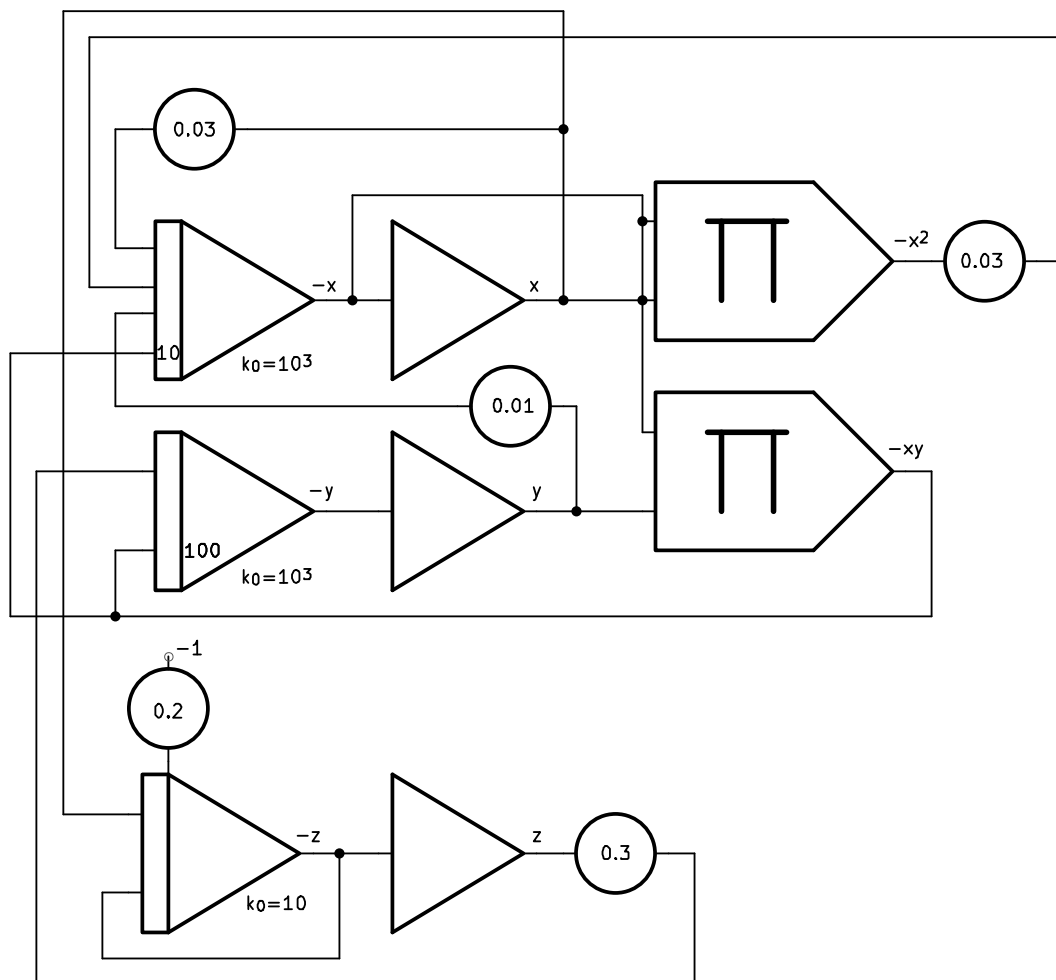


Figure 1: Analog computer program for equations (1), (2), (3)

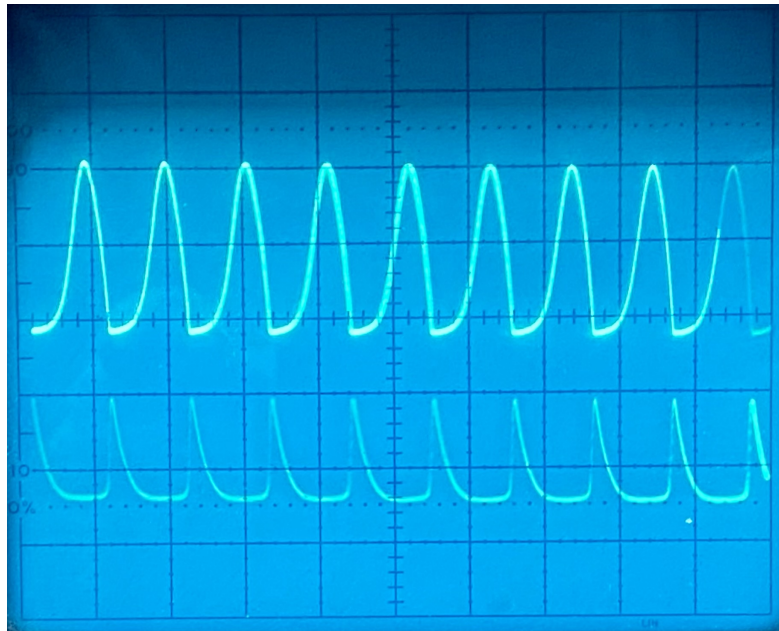


Figure 2: Behavior of the system

```
#include "THAThc.h"
#define BAUD_RATE 250000

void setup() {
  Serial.begin(BAUD_RATE);
  THAThc.begin();
}

void loop() {
  THAThc.shell();
}
```

Figure 3: Arduino control program

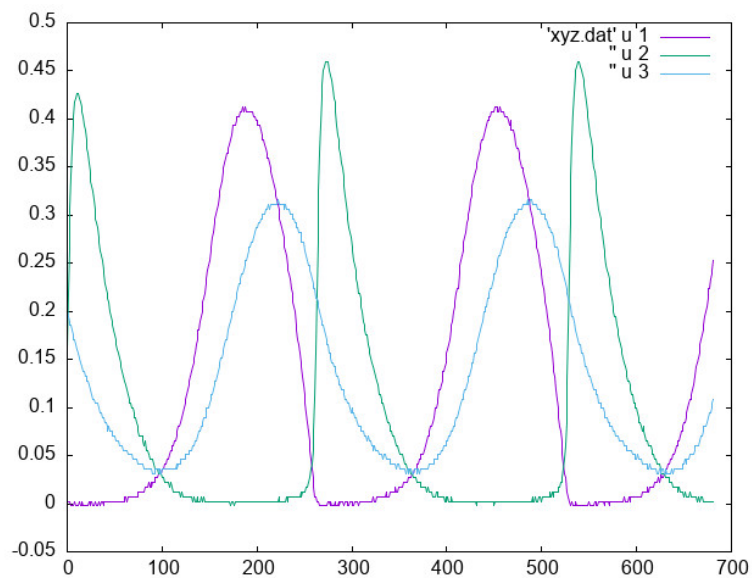


Figure 4: x -, y -, and z -channels captured using the Arduino based hybrid controller

```
arm
run
read
```

The `read` command will dump the values of all three channels over the serial line. They then can be copied from the serial monitor and pasted into a file for subsequent processing. Figure 4 shows a plot generated using `gnuplot`.

Happy analog computing!



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References

- [FIELD et al. 1974] RICHARD J. FIELD, RICHARD M. NOYES, "Oscillations in chemical systems. IV. Limit cycle behavior in a model of a real chemical reaction", in *The Journal of Chemical Physics*, Vol. 60, No. 5, 1 March 1974, pp. 1877–1884
- [TYSON 1981] JOHN J. TYSON, "On Scaling the Oregonator Equations", in [VIDAL et al. 1981, pp. 222 ff.]
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