



Time derivatives

1 Computing time derivatives

In some cases it might be necessary to generate a time derivative of some signal on an analog computer. It should be noted that in general integration should always be favored over differentiation as the latter tends to increase noise substantially. However, when one wants to implement e. g. a PID-controller, the proportional (P) and integral (I) part are easy but the derivative (D) turns out to be a bit tricky. In this application note two approaches to compute approximate solutions for $\frac{d}{dt}f(t)$ are shown.¹

2 A simple signal generator

First, a simple signal generator is described. Its output signals serve as simple, yet suitable test inputs to the differentiator circuits. This circuit yields a cosine-, a square-, and a triangular signal as shown in figure 1. The schematic is straightforward: The upper subcircuit generates a $\cos(\omega t)$ signal using two Z-diodes to prevent an overload.² This signal controls a comparator switching between $+1$ and -1 yielding a square wave signal. This signal is then fed into a third integrator yielding a

¹More details on the techniques described below can be found in [GILOI et al. 1963, pp. 160 ff.], [SYDOW 1964, pp. 46 f.], and [ULMANN 2023/1, pp. 89 ff.].

²In fact, both integrators, especially the second one, will still experience an overload but not so much as to distort the signal too much.

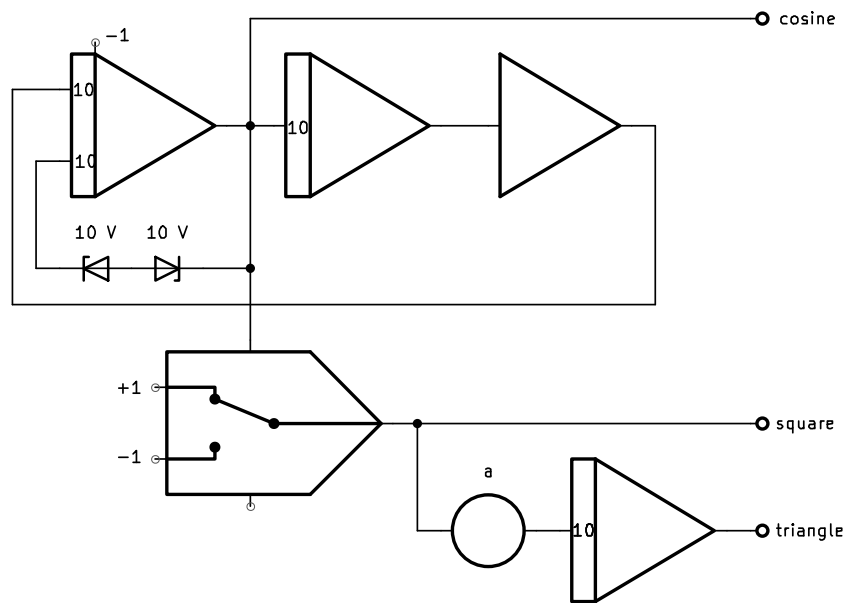


Figure 1: Simple signal generator

triangular signal.³ The coefficient a should be set to get a nice and large triangular signal without overload.⁴

3 Direct time derivative

The first circuit shown is pretty straightforward: The basic idea is to swap the input resistor and feedback capacitor of an integrator as shown in figure 2. The capacitor and the coefficient b are chosen manually to

³Although this circuit is far from perfect, the integrator yielding the triangular output signal is stable enough to allow for run times of several minutes before drifting away too far in the case of a THAT.

⁴In systems which do not have comparators, an external signal generator should be used.



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yield the best possible time derivative for a given signal type. Picture 3 shows the behavior of this setup.⁵

4 A better circuit

A much better implementation of a time derivative which does not require additional passive elements and works really well on modern systems is shown in figure 4. The parameter α is of central importance, controlling the frequency response of the circuit and should be as large as possible without causing overshoots or instabilities in the output signal of the circuit. In typical setups $\alpha \approx 0.85$. The behavior of this circuit is shown in figure 5.⁶ The weight of 10 at the integrator input is due to the frequency of the signal generated by the simple circuit shown in figure 1. At even higher frequencies even larger weights might be necessary.

A rather handwavy explanation of the circuit goes like this: The summer, followed by an inverter and a (rather large) positive feedback by the coefficient α approximates a non-inverting open amplifier with the sign flipping integrator in its main feedback loop. It will thus yield an output signal $y(t)$ satisfying $\int y(t) dt = f(t)$ with $f(t)$ denoting the input signal. The only problem here is the instability of the algebraic loop consisting of the summer and inverter when α becomes too large. From a mathematical point of view $\alpha \rightarrow 1$ would be desirable. Practical

⁵The quality of the output signal could be improved by adding a small series resistor between the capacitor and the summing junction of the inverter/summer. The screen shots were taken from a Gould OS4200 digital storage oscilloscope which explains the rather uneven exposure of the pictures.

⁶This circuit can be used directly as the D-part of a PID controller.

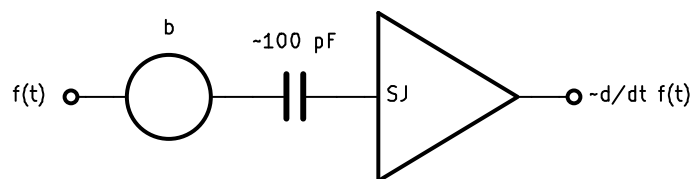


Figure 2: Direct differentiation circuit

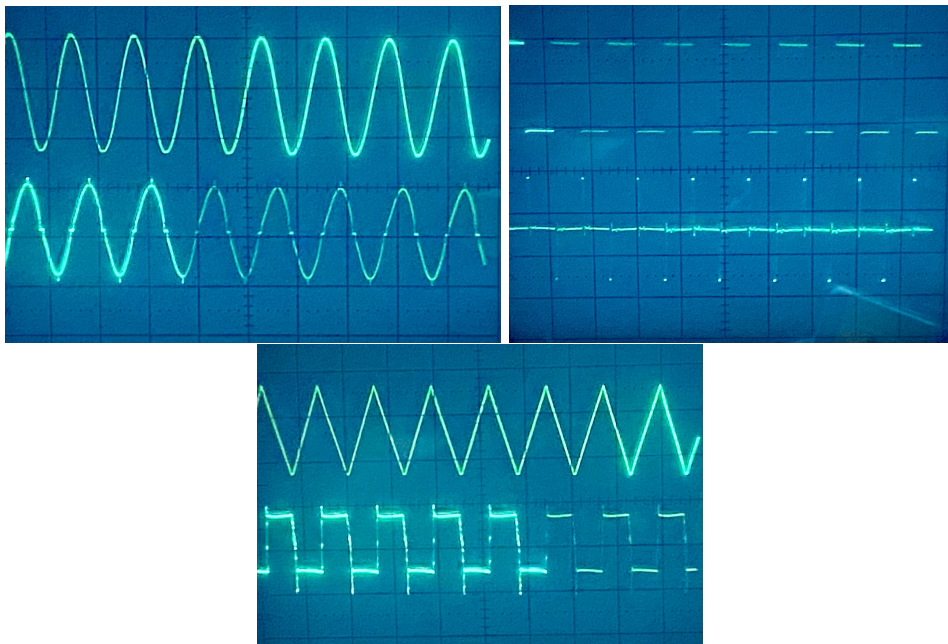


Figure 3: Behavior of the direct differentiation circuit applied to a cosine, a square wave, and a triangular signal



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experiments show that the circuit tends to get instable for $\alpha > 0.85$, though.

A more thorough explanation takes a look at the output signal $y(t)$ at the inverter:

$$y(t) = f(t) - \int y(t) dt + \alpha y(t)$$

Taking the time derivative on both sides yields

$$\dot{y}(t) = \dot{f}(t) - y(t) + \alpha \dot{y}(t).$$

Bringing $\dot{y}(t)$ to the right and $y(t)$ to the left then yields

$$\begin{aligned} y(t) &= \dot{f}(t) - \dot{y}(t) + \alpha \dot{y}(t) \\ &= \dot{f}(t) + (\alpha - 1)\dot{y}(t). \end{aligned}$$

Now let $\alpha \rightarrow 1$, so that $y(t) \approx -\dot{f}(t)$.

References

- [GILOI et al. 1963] WOLFGANG GILOI, RUDOLF LAUBER, *Analogrechnen*, Springer-Verlag, 1963
- [SYDOW 1964] ACHIM SYDOW, *Programmierungstechnik für elektronische Analogrechner*, VEB Verlag Technik Berlin, 1964
- [ULMANN 2023/1] BERND ULMANN, *Analog and Hybrid Computer Programming*, 2nd edition, DeGruyter, 2023

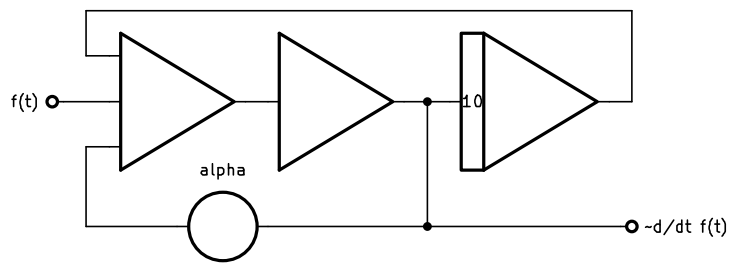


Figure 4: Clever time derivative circuit

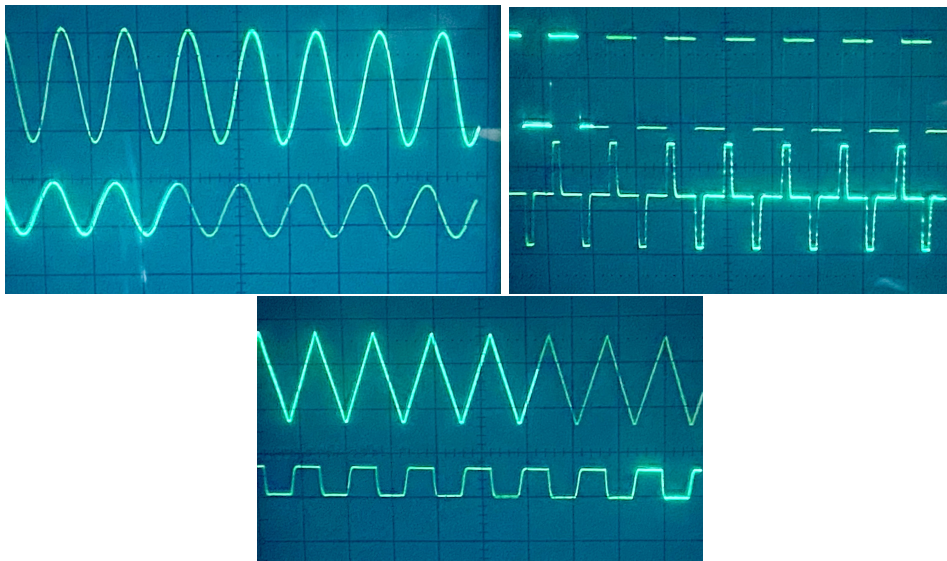


Figure 5: Behavior of the differentiation circuit shown in figure 4 applied to a cosine, a square wave, and a triangular signal