

Zombie Apocalypse Now¹

1 Introduction

How could one not like zombie movies? It was about time that someone, ROBERT SMITH? and his collaborators, shed some light on zombie attacks from a mathematical point of view (see [SMITH?(2014)]). PHIL MUNZ (see [MUNZ(2014)]) applied a well-known coupled set of differential equations due to ALFRED JAMES LOTKA and VITO VOLTERRA in the late 19th/early 20th century to this particular problem. Although VOLTERRA and LOTKA were interested in (closed) eco-systems, their equations are ideally suited to model a world infested by zombies. Whenever there is a problem which is readily described by (coupled) differential equations, an analog computer is the ideal tool to tackle them as will be done in the following.

2 Programming

The mathematical model itself is quite straightforward and consists of the following two coupled differential equations:

$$\frac{dh}{dt} = \alpha h - \beta h z \quad (1)$$

$$\frac{dz}{dt} = \delta h z - \gamma h z - \zeta z \quad (2)$$

¹Version 1.2

h and z represent the number of humans and zombies, respectively. h_0 and z_0 are the initial conditions. The remaining parameters are

α : Growth rate of the human population (birthrate).

β : Factor describing the rate at which humans are killed by zombies.

δ : Growth factor of zombie population due to zombies killing humans and thus transforming them into zombies.

γ : Rate at which zombies are killed by humans.

ζ : Normal “death” rate of the zombie population.

Figure 1 shows the resulting program for the two equations (1) and (2) while figure 2 shows the program as implemented on an Analog Paradigm Model-1 analog computer.

3 Results

Figure 3 shows a typical result obtained with the setup described above. The computer was run in repetitive mode with the time-constants of the integrators set to $k_0 = 10^3$. Additionally, only integrator inputs with a weight of 10 were used, further speeding up the simulation by another factor of 10. The IC-time was set to short, and the OP-time to 60 ms. The oscilloscope was explicitly triggered with one of the TRIG-outputs of the CU to obtain a stable, flicker-free display.

The parameters were derived experimentally by playing with the various coefficients until a oscillatory behaviour was obtained. The output

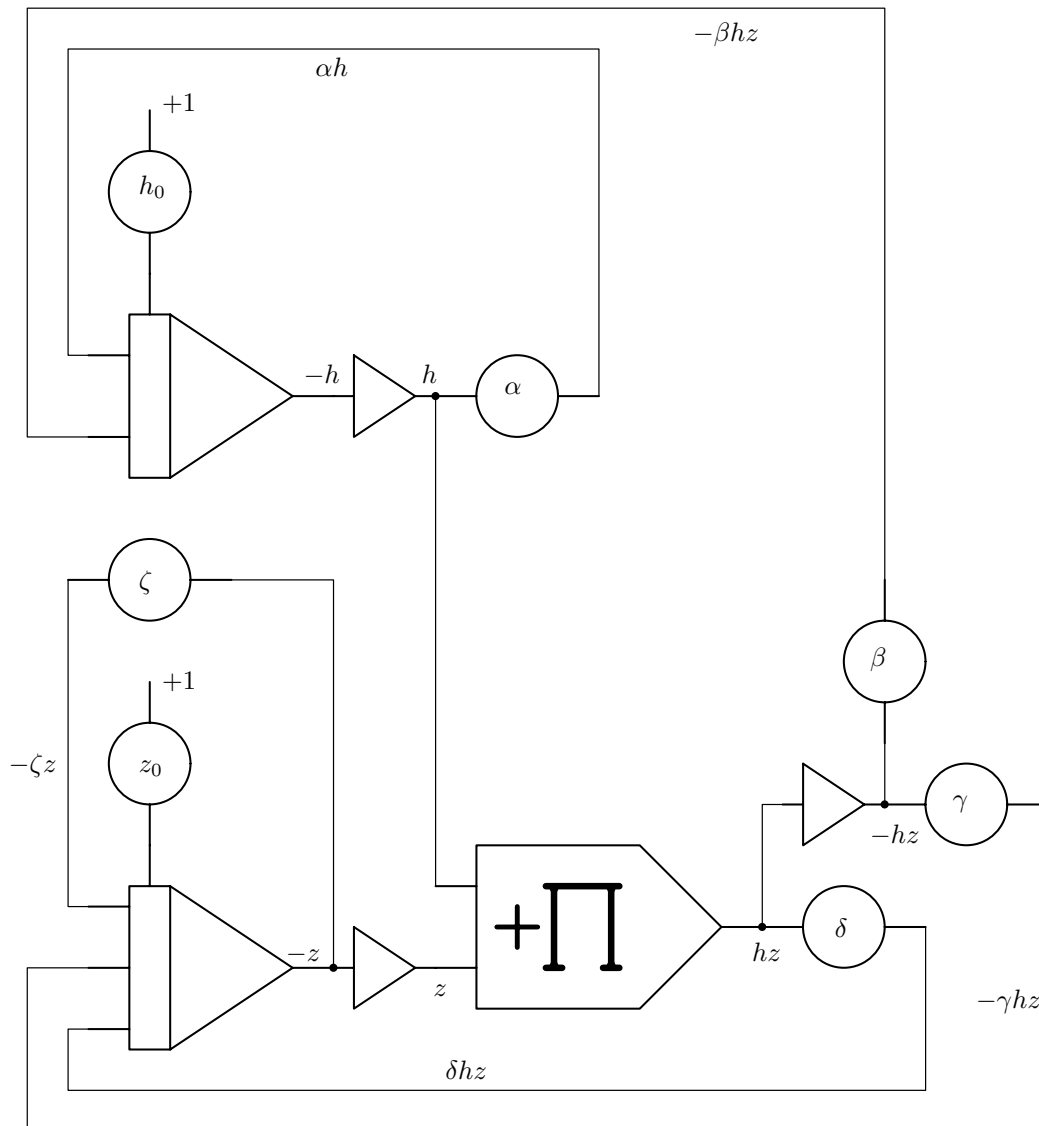


Figure 1: Analog computer program for equations (1) and (2)

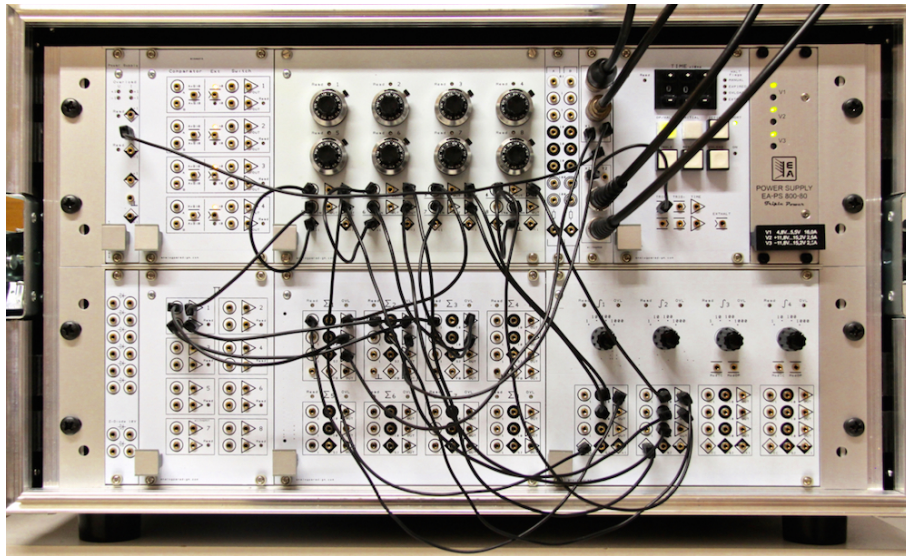


Figure 2: Zombie simulation on an Analog Paradigm Model-1 analog computer

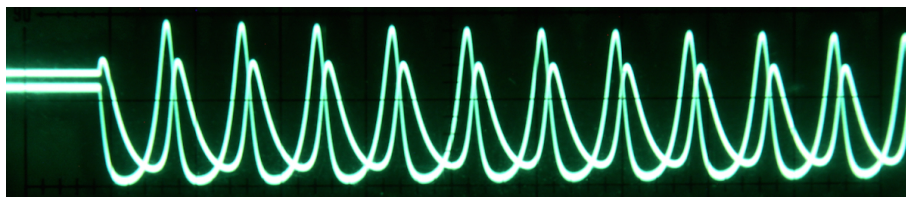
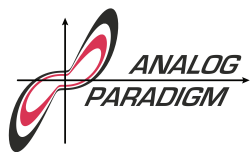


Figure 3: Results of a typical zombie simulation



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shown was generated with $h_0 = z_0 = 0.6$, $\alpha = 0.365$, $\beta = 0.95$, $\delta = 0.84$ (very successful zombies, indeed), $\gamma = 0.44$, and $\zeta = 0.09$.

References

- [SMITH?(2014)] ROBERT SMITH?, *Mathematical Modelling of Zombies*, University of Ottawa Press, 2014
- [MUNZ(2014)] PHIL MUNZ, "When Zombies Attack! Alternate Ending", in [SMITH?(2014)], pp. 45–55