

The LORENZ 96 model

1 Introduction

In 1996, EDWARD NORTON LORENZ¹ of “LORENZ attractor fame”² described another chaotic system, now known as the *LORENZ 96 model* which consists of $N \in \mathbb{N}, N \geq 4$ coupled differential equations of the form:³

$$\dot{x}_i = (x_{i+1} - x_{i-2})x_{i-1} - x_i + F \quad (1)$$

with periodic “boundary conditions” (so to speak)

$$x_0 = x_N,$$

$$x_{-1} = x_{N-1}, \text{ and}$$

$$x_{N+1} = x_1$$

and a forcing constant F which is typically set to $F = 8$. All equations (1) can be scaled with a common factor $\lambda = \frac{1}{20}$, yielding

$$\dot{x}_0 = 20(x_1 - x_2)x_3 - x_0 + F^*, \quad (2)$$

$$\dot{x}_1 = 20(x_2 - x_3)x_0 - x_1 + F^*, \quad (3)$$

$$\dot{x}_2 = 20(x_3 - x_0)x_1 - x_2 + F^*, \text{ and} \quad (4)$$

$$\dot{x}_3 = 20(x_0 - x_1)x_2 - x_3 + F^* \quad (5)$$

for $N = 4$ with a scaled forcing constant F^* .

¹23.05.1917–16.04.2008

²See application note #2 from 13.09.2016, https://analogparadigm.com/downloads/alpaca_2.pdf.

³See [LORENZ 1996, pp. 4 f.] and [STERK et al. 2017].

Analog values					
point attractor		cyclic attractor		chaotic attractor	
$F^* \geq$	$F^* \leq$	$F^* \geq$	$F^* \leq$	$F^* \geq$	$F^* \leq$
0	0.051	0.052	0.627	0.627	0.77

Table 1: Influence of F^*

2 Implementation

The implementation of equations (2), (3), (4), and (5) is straightforward and shown in figure 1.

3 Results

The system shows interesting behavior for varying F^* , with three attractor types shown in table 1.

Figures 2, 3, 4, and 5 show the behavior of the system for various values of F^* . On the right are phase space plots with an FFT analysis on the left. The switch from periodic to chaotic behavior can be clearly seen in the spectrum which starts to resemble more and more that of a noise signal.

The variation of the forcing constant F^* can be automated by integrating over a suitably small constant c such as $c = -0.2$ yielding a linear ramp $F^*(t)$ running from 0 to 1 in 5 seconds with the integrator set to $k_0 = 1$. Plotting $x(t)$ against $F^*(t)$ yields a simple bifurcation plot of the LORENZ 96 system as shown in figure 6.

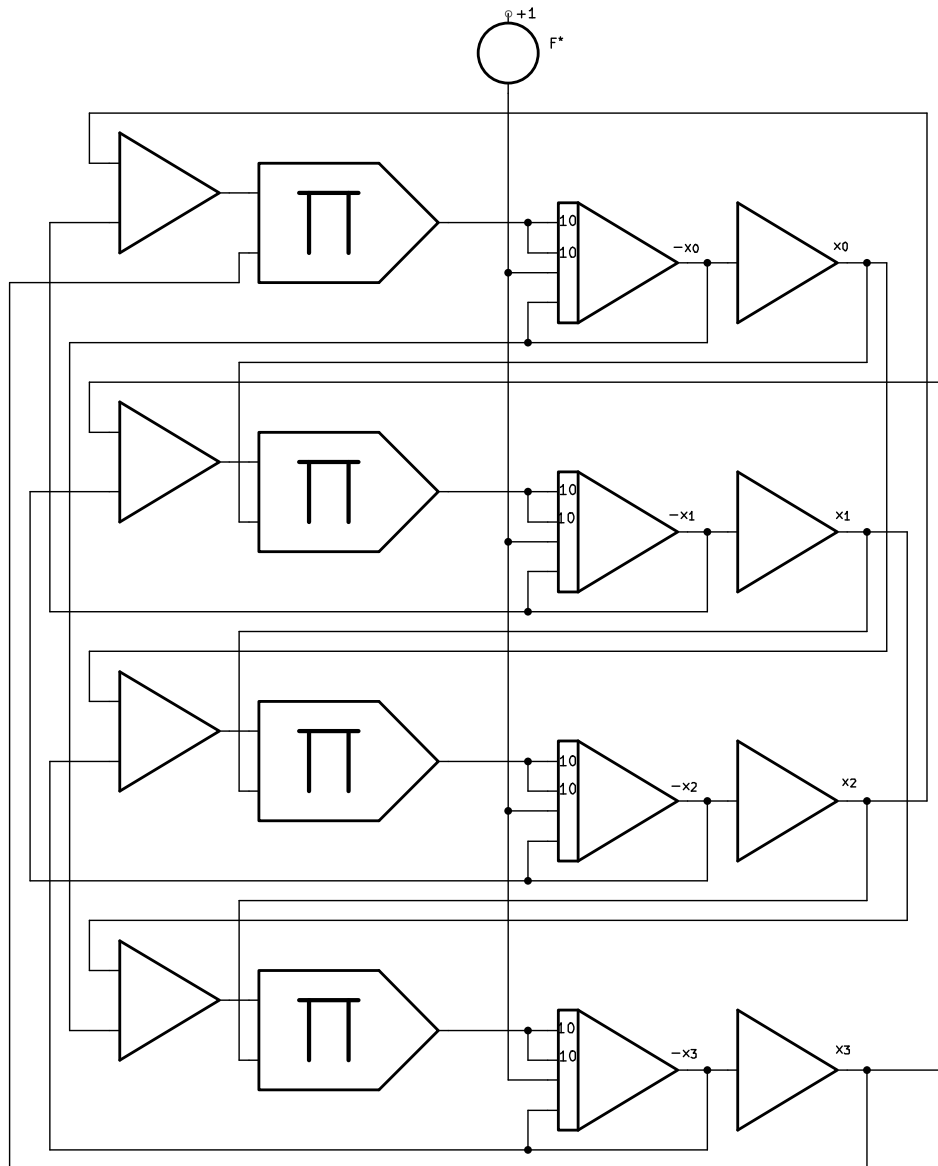


Figure 1: Implementation of the scaled LORENZ 96 system with $N = 4$

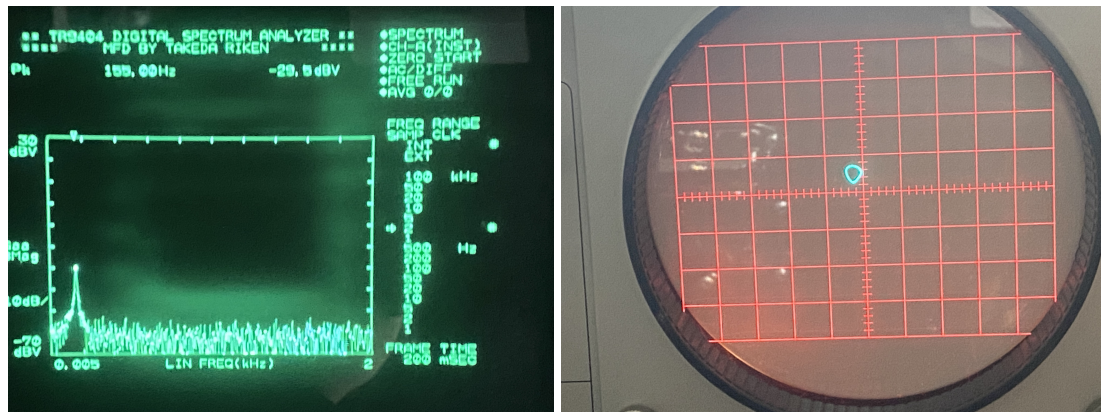


Figure 2: Start of the periodic attractor

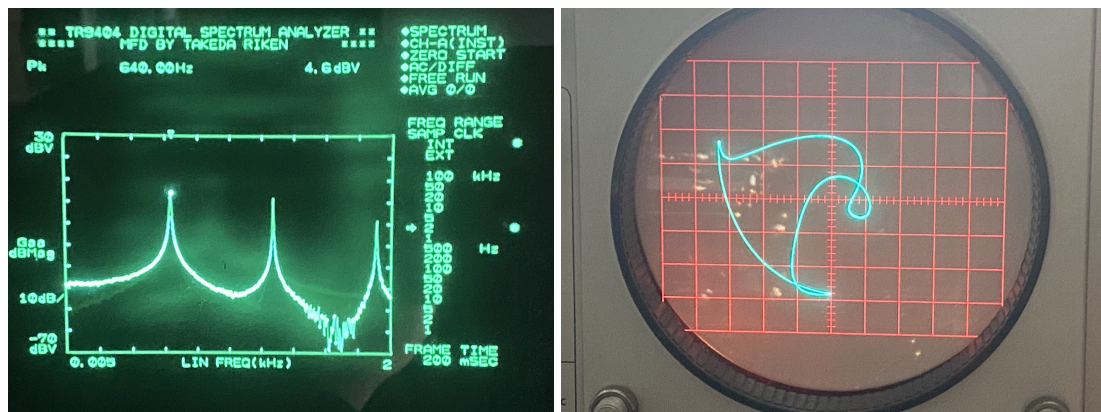


Figure 3: End of the periodic attractor

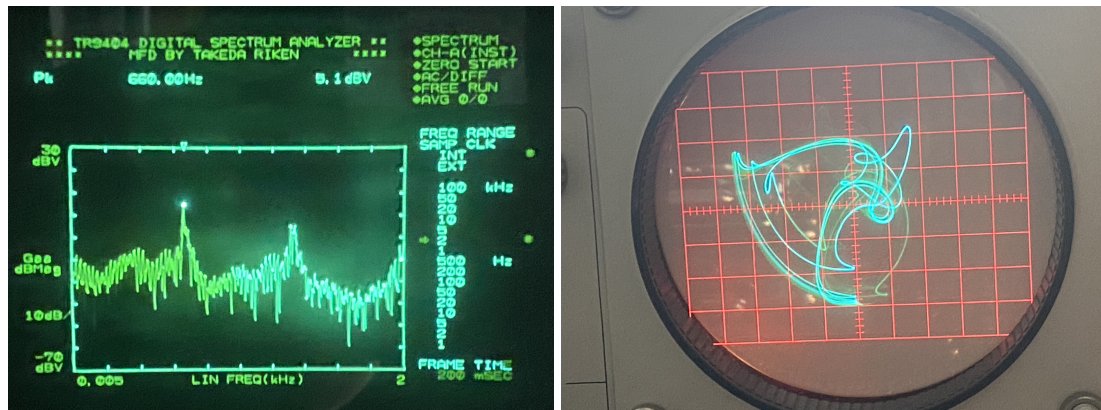


Figure 4: Start of the chaotic attractor

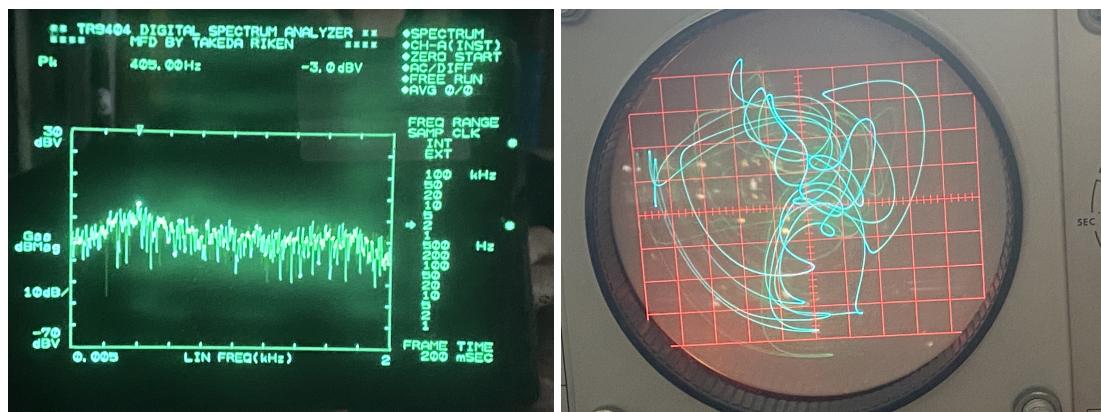


Figure 5: End of the chaotic attractor (maximum F^*)

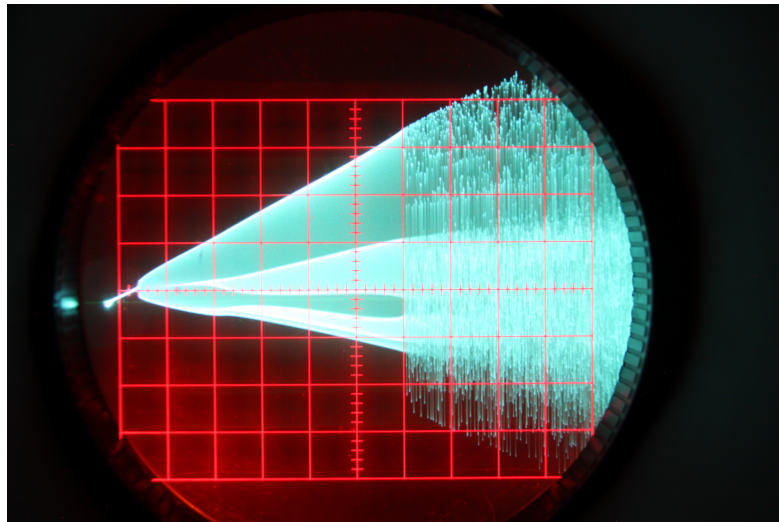


Figure 6: Simple bifurcation diagram of the LORENZ 96 system

References

- [LORENZ 1996] EDWARD NORTON LORENZ, "Predictability – a Problem Partly Solved", in *Proceedings on Seminar on Predictability*, 1, pp. 1–18
- [STERK et al. 2017] A. E. STERK, D. L. VAN KEKEM, "Predictability of Extreme Waves in the Lorenz-96 Model Near Intermittency and Quasi-Periodicity", in *Complexity*, 2017, Article ID 9419024, <https://doi.org/10.1155/2017/9419024>