

A velocity forced oscillator

I know, I know. . . Yet another chaotic system – can't you think of something else? Honestly? No. . . :-) At least not at the moment. The more I play with chaotic systems the more intrigued I am, thus this flood of chaos-oriented application notes.

While reading JOOST REKVELD's wonderful book *Liberate the Machines!*¹ I found phase space plots of his implementation of a chaotic system (p. 103) due to [SPROTT 2010, pp. 53 ff]. The system under consideration is a *velocity forced oscillator*, i.e., an oscillatory system under the influence of a harmonic forcing function in its velocity term. Such systems are of the general structure

$$\begin{aligned}\dot{x} &= v + A \sin(\omega t) \text{ and} \\ \dot{v} &= f(v, x).\end{aligned}$$

The particular system described in the following is SPROTT's VF2 which gives rise to a particularly beautiful phase space plot. It is described by

$$\dot{v} = (1 - x^2)v - 0.2x$$

with $v = \dot{x}$ (and $A = \omega = 1$). The corresponding system of coupled differential equations is thus

$$\begin{aligned}\dot{v} &= wv - 0.2x, \\ \dot{x} &= v + F, \text{ and} \\ w &= 1 - x^2\end{aligned}$$

¹See [REKVELD 2025].

with a sinusoidal forcing function generated by $\ddot{F} = -F$ with $\dot{F}(0) = -1$. Suitable scaling factors are $\lambda_v = \lambda_x = \frac{1}{5}$ and $\lambda_w = \frac{1}{25}$ yielding

$$\begin{aligned} \dot{v} &= 25wv - 0.2x, \\ \dot{x} &= v + 0.2F, \\ w &= 0.04 - x^2, \text{ and} \\ \ddot{F} &= -F. \end{aligned} \tag{1}$$

The resulting analog computer program is straightforward and shown in figure 1. Interestingly, the parameters of equation (1) must be tweaked a bit to yield a result comparable to that given in [SPROTT 2010, p. 54]:

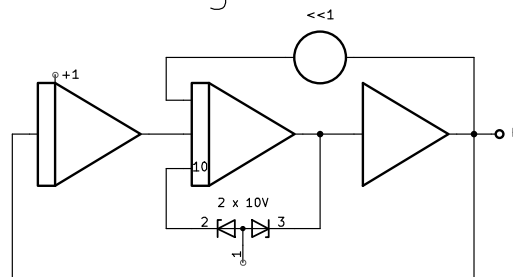
$$\dot{v} = 26wv - 0.18x$$

Generation of the forcing function is straightforward. The potentiometer generating some positive feedback on the second integrator from the left “fans” the oscillator. Its value should be as small as possible, in most cases it can be omitted. The two 10 V Z-diodes limit the output amplitude. Using an input with weight 10 instead of the summing junction (as typically suggested in the literature) results in a cleaner spectral output. The phase space plot shown in figure 2 plots $-v$ against $-x$.

References

- [REKVELD 2025] JOOST REKVELD, *Liberate the Machines!*, Studio Joost Rekveld Brussels – Kask & Conservatorium Ghent, 2025
- [SPROTT 2010] JULIEN CLINTON SPROTT, *Elegant Chaos – Algebraically Simple Chaotic Flows*, World Scientific, 2010

Forcing Function



Sprott VF2

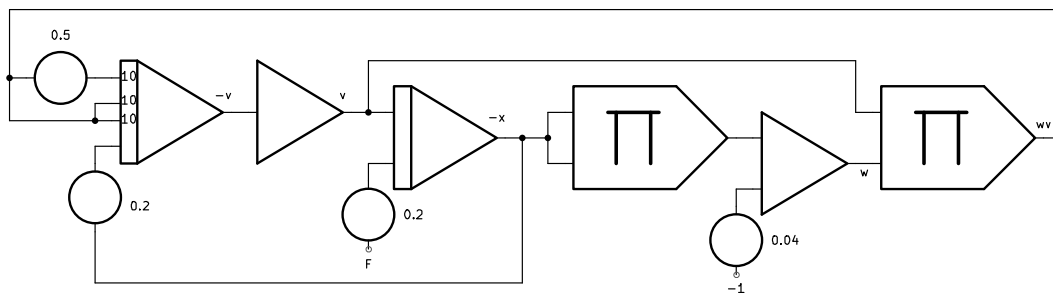


Figure 1: Implementation of SPROTT's VF2 system

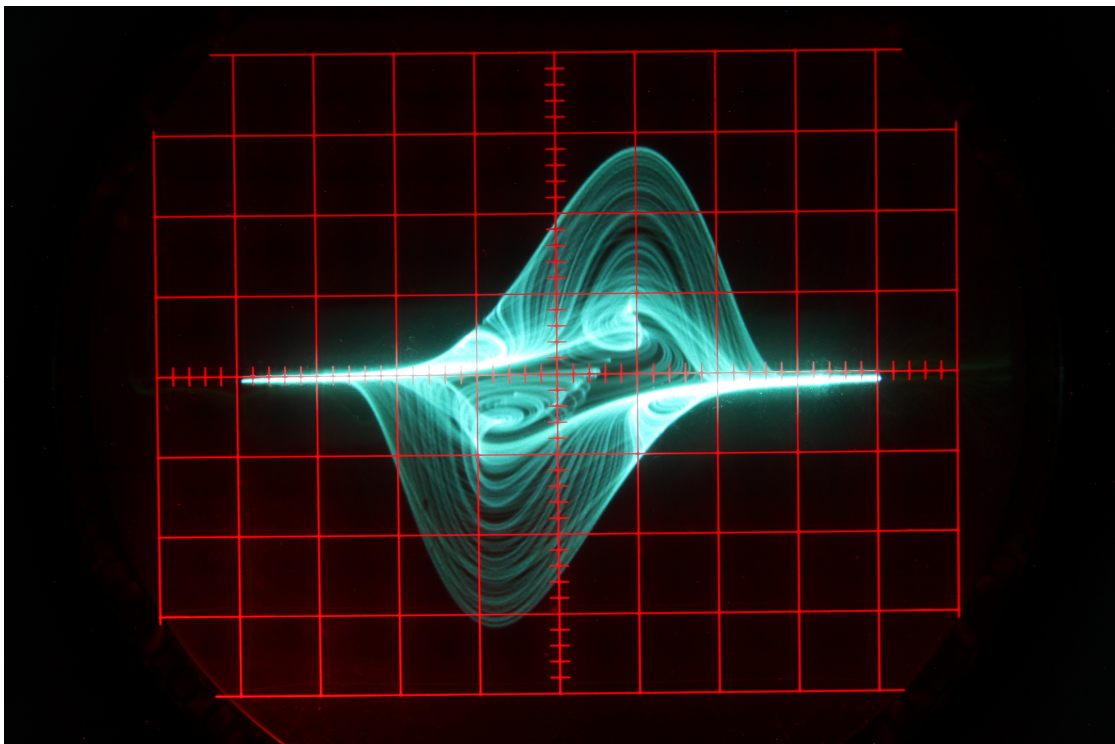


Figure 2: Phase space plot of the VF2 system