Reminiscing about COMPENG'26

1 Introduction

From July 8^{th} to July 10^{th}, 2026, COMPENG'26 took place at beautiful Castel Ivano,[1] Italy. Although all of the talks given there were very interesting, two were of special interest to the author (who would like to explicitly thank Prof. Dr. Ludovico Minati for the invitation to this truly wonderful conference) and inspired this little application note.

2 Chaotic system with one stable equilibrium

In the very first talk of the conference, *From the Lorenz system to non-hyperbolic chaotic systems*, Guanrong Chen showed a number of highly interesting dynamical systems, particularly the following one with one stable equilibrium:[2]

$$\begin{aligned}\dot{x} &= yz + a\\ \dot{y} &= x^2 - y\\ \dot{z} &= 1 - 4x\end{aligned}$$

Here, a (being small) denotes the control parameter. This can be easily implemented on THE ANALOG THING (*THAT* for short) as shown in

[1] See `https://www.castelivano.it/en/home-en/`, retrieved 11.07.2026.
[2] See [Wang et al.(2012)].

the following: Suitable scaling factors for this system are[3] $\lambda_x = \lambda_y = \frac{1}{3}$ and $\lambda_z = \frac{1}{8}$, yielding

$$\begin{aligned}\dot{\widehat{x}} &= 8\widehat{y}\widehat{z} + \widehat{a},\\ \dot{\widehat{y}} &= 3\widehat{x}^2 - \widehat{y}, \text{ and}\\ \dot{\widehat{z}} &= 0.125 - 1.5\widehat{x}\end{aligned}$$

with $\widehat{a} = 0.333a$. Experiments have shown $\widehat{a} \approx 0.007$ to be a good value. The resulting schematic and the actual implementation are shown in figures 1 and 2[4] while three typical phase space plots are shown in figure 3.

3 Variation of the Chua-system

In a later talk given by Riccardo Meucci a variant of the well-known Chua-system[5], the Ginoux-Meucci-Chua-system (*GMC-system*), was shown. This, too, can be implemented on a single THAT as shown in the following.

This system is described by the following set of equations and differs only slightly from the original Chua-system:

[3] As it turns out, especially in conjunction with chaotic systems, more conservative scaling factors should be preferred over too aggressive ones, i. e., it is better to sacrifice a (small) portion of the interval $[-1, 1]$ in order to have some "wiggle-room" for variables behaving unexpectedly.

[4] The three inverters to derive x, y, and z are not really necessary since $x^2 = (-x)^2$ and $yz = (-y)(-z)$, but it is nice to have all variables with both signs for making phase space plots.

[5] See [Ulmann(2016)].

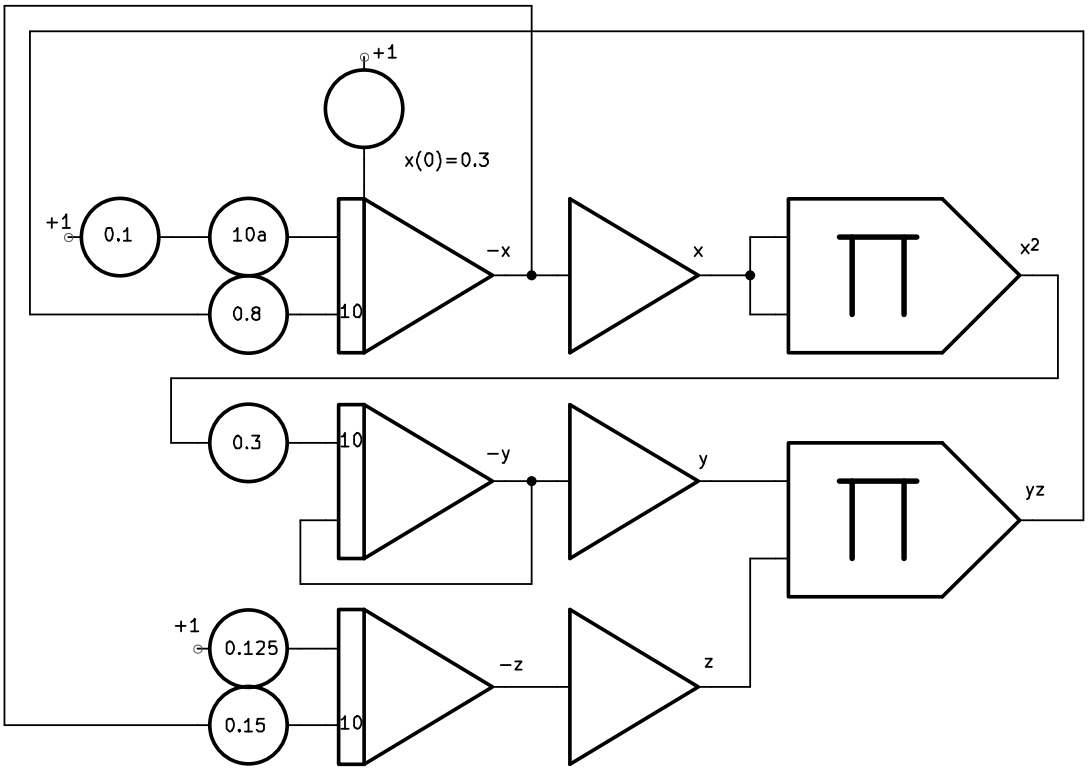


Figure 1: Analog computer program of the chaotic system with one stable equilibrium

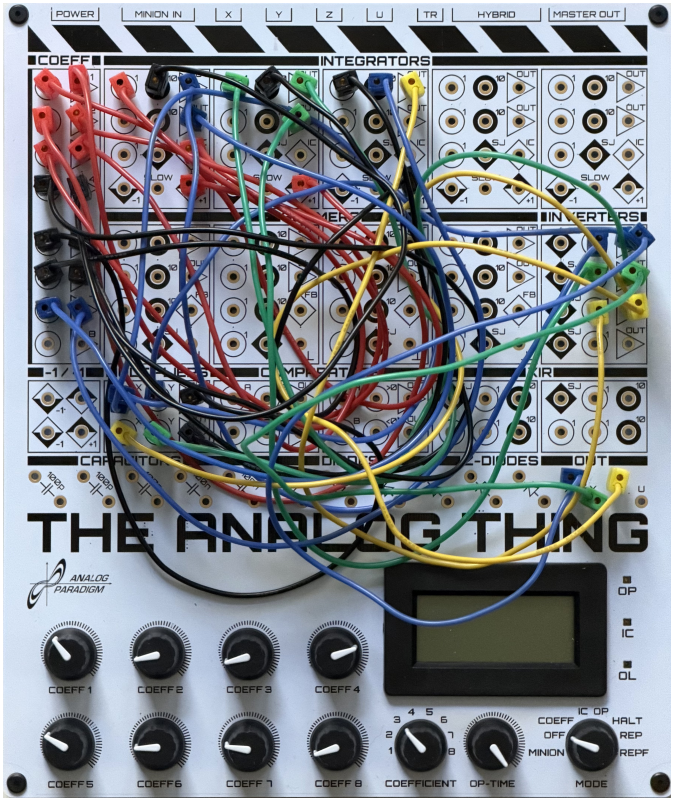


Figure 2: Implementation of the chaotic system with one stable equilibrium on THE ANALOG THING

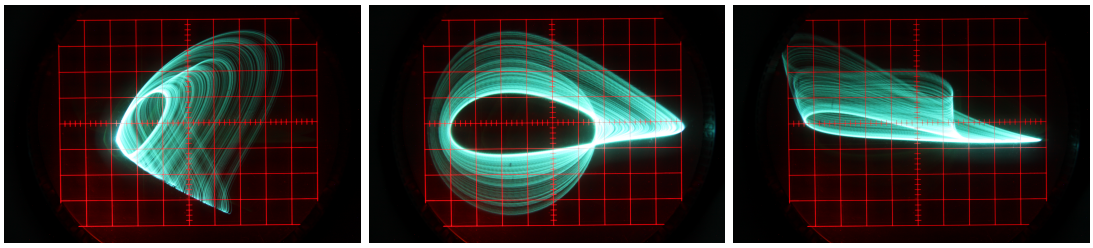


Figure 3: Phase space plots of the chaotic system with one stable equilibrium

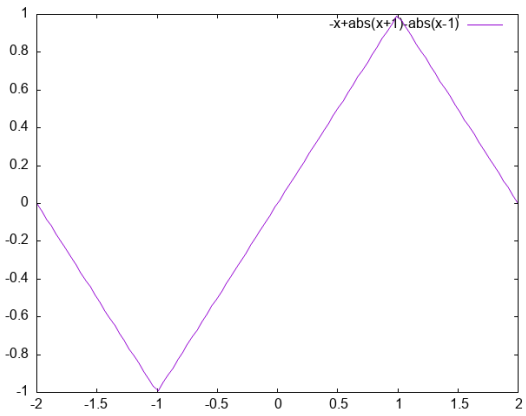


Figure 4: $f(x)$ for $x \in [-2, 2]$

$$\dot{x} = -\alpha(y - f(x)) = -\alpha y + \alpha f(x), \tag{1}$$
$$\dot{y} = -x + z, \text{ and} \tag{2}$$
$$\dot{z} = -\beta y \tag{3}$$

with $f(x)$ being the piecewise linear function

$$f(x) = bx + \frac{a-b}{2}\left(|x+1| - |x-1|\right).$$

Here $\alpha = 0.4$, $a = 1$, $b = -1$, and $\beta \approx 0.95$ is the control parameter. With a and b set to these values, $f(x)$ becomes

$$f(x) = -x + \left(|x+1| - |x-1|\right).$$

Its behavor is shown in figure 4.

A quick numeric experiment yields the following scaling factors for x, y, and z: $\lambda_x = \lambda_y = \frac{1}{6}$ and $\lambda_z = \frac{1}{10}$. Ignoring the absolute value

functions for now, equations (1), (2), and (3) are thus transformed into

$$\begin{aligned}\dot{\widehat{x}} &= -\alpha\widehat{y} + 0.1666\alpha\widehat{f}(\widehat{x}), \\ \dot{\widehat{y}} &= -\widehat{x} + 1.666\widehat{z}, \text{ and} \\ \dot{\widehat{z}} &= -\widehat{\beta}\widehat{y}\end{aligned} \tag{4}$$

with $\widehat{\beta} = 0.6\beta = 0.57$ and

$$\widehat{f}(\widehat{x}) = -(6\widehat{x} - |6\widehat{x} + 1| + |6\widehat{x} - 1|).$$

The arguments of the absolute value functions now also have to be scaled by λ_x, otherwise an overload in the summation would occur. This yields

$$\widehat{f}(\widehat{x}) = -6(\widehat{x} - |\widehat{x} + 1| + |\widehat{x} - 1|).$$

Now $\widehat{f}(\widehat{x})$ has to be scaled down again. Since the absolute value functions basically counteract each other, a scaling factor of $\lambda_{\widehat{f}(\widehat{x}} = \frac{1}{2}$ would be sufficient, however, due to the limited amount of coefficients available on a THAT it will just scaled by $\lambda_{\widehat{f}(\widehat{x})} = \frac{1}{6}$ so that many factors will just cancel out, especially that factor 0.1666 in equation (4), yielding the final system of equations

$$\dot{\widehat{x}} = -\alpha y + \alpha\widehat{f}(\widehat{x}), \tag{5}$$

$$\dot{\widehat{y}} = -x + 1.666\widehat{z}, \tag{6}$$

$$\dot{\widehat{z}} = -\widehat{\beta}y, \text{ and} \tag{7}$$

$$\widehat{f}(\widehat{x}) = -(\widehat{x} - |\widehat{x} + 0.1666| + |\widehat{x} - 0.1666|). \tag{8}$$

The classic approach to implement an absolute value function on an analog computer is shown in figure 5 (see [Ulmann(2023), pp. 82 f.]).

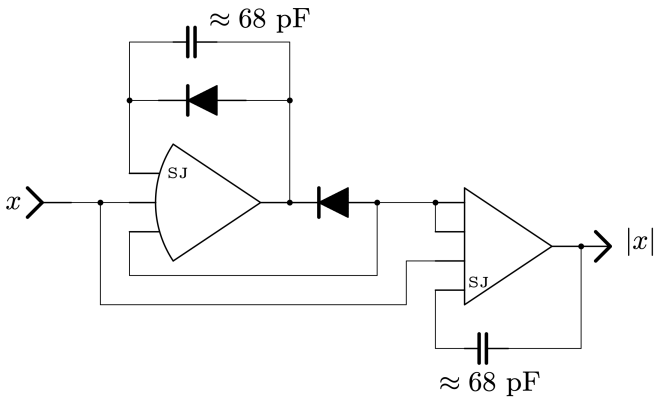


Figure 5: Standard circuit for an absolute value function

This requires two summers per function, one configured as an open amplifier.[6] The two sums $\widehat{x} \pm 0.1666$ in equation (8) each require an additional summer, thus eventually exceeding the computing elements of a single THE ANALOG THING.

The main circuit implementing equations (5), (6), and (7) is shown in figure 6.[7] Since the classic absolute value function implementation, required twice for this problem, is not feasible, a different approach using two comparators is used to implement $f(x)$ as shown in figure 7. This circuit fits nicely onto a single THE ANALOG THING as shown in figure 8. Since one summer is unused, the second potentiometer in figure 7 can be omitted and the input to the summer fed with the inverted value of the corresponding input of the summer on the top left. The beautiful double-scroll-attractor obtained by this circuit is shown in figure 9.

[6] This basically implements an ideal diode.
[7] The hats over the scaled variables and functions are omitted in the following to simplify notation.

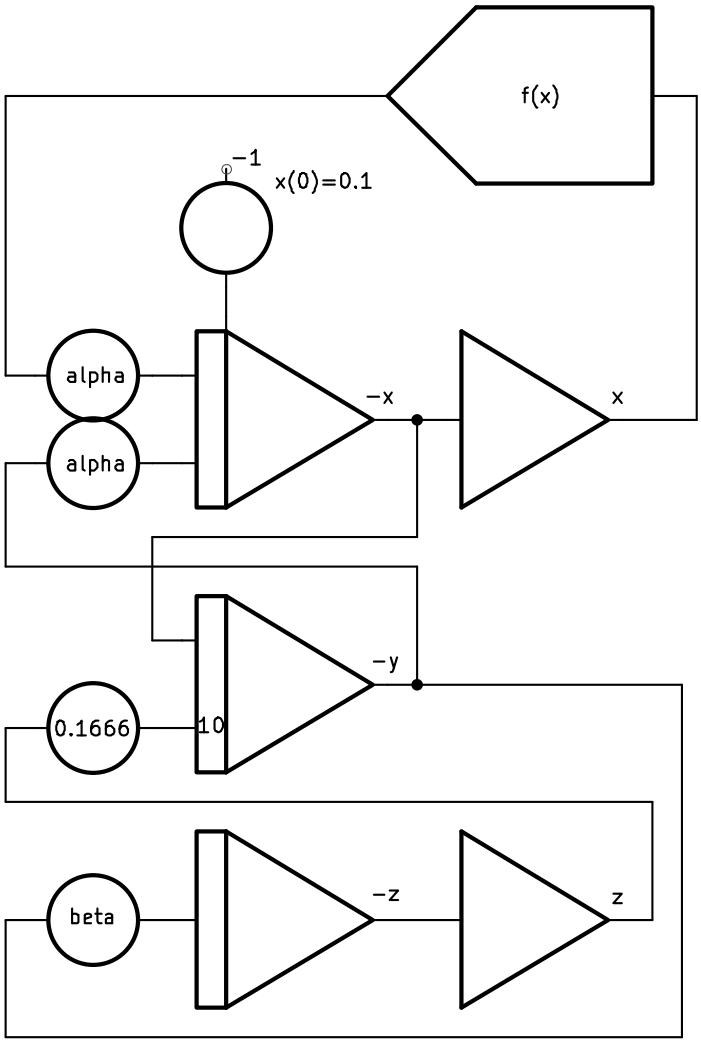


Figure 6: Circuit implementing equations (5), (6), and (7)

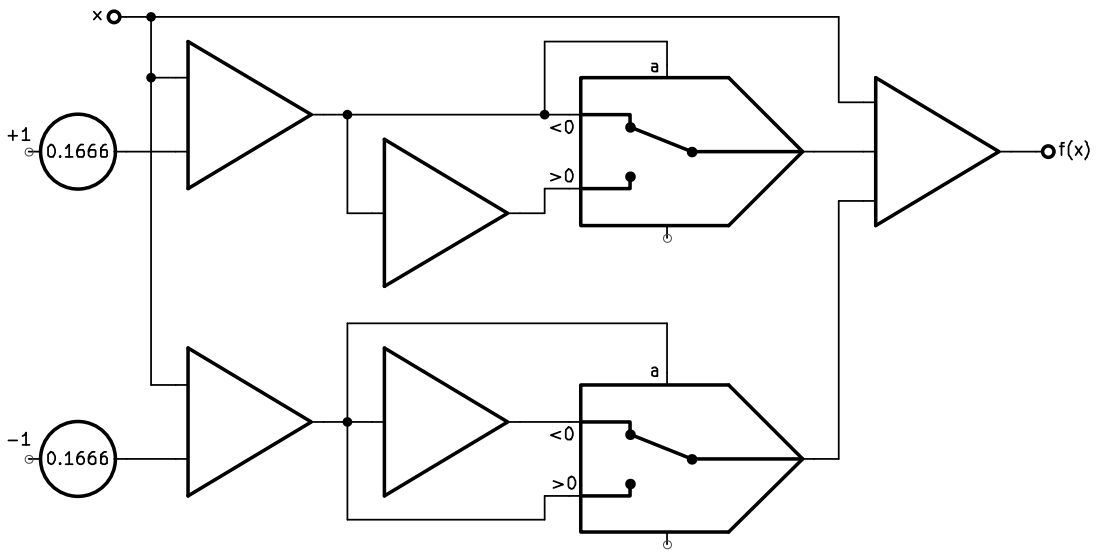


Figure 7: Circuit implementing equation (8)

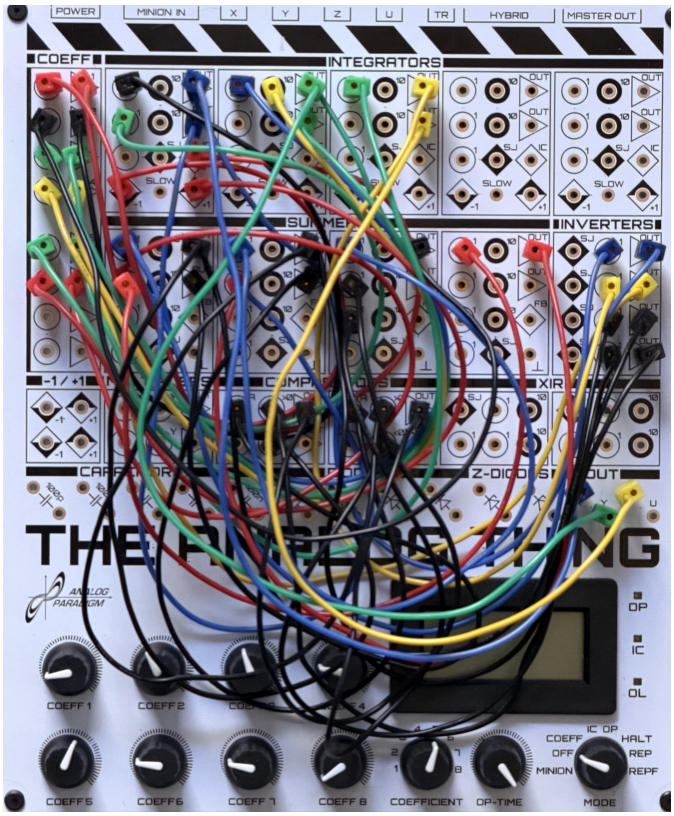


Figure 8: Actual implementation of the circuits shown in figures 6 and 7

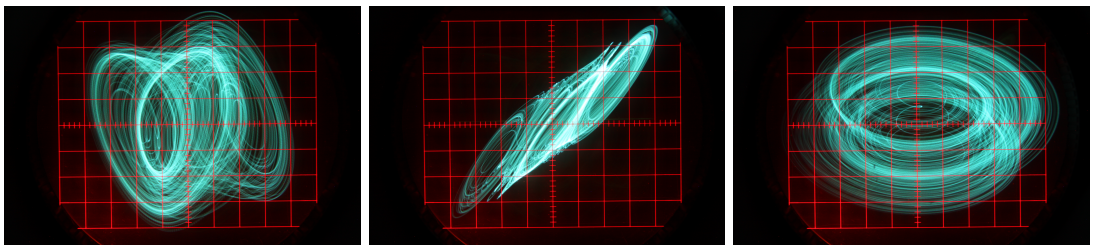


Figure 9: Double-scroll-attractor of the GMC-system

4 A cubic system

A variation of this system can be found in [Ginoux et al.(2025), p. 7162] where the piecewise linear function is replaced with a much simpler cubic term:

$$\begin{aligned}\dot{x} &= -\alpha y + \alpha x - \alpha x^3\\ \dot{y} &= -x + z\\ \dot{z} &= -y\end{aligned}$$

Here, $\alpha = 0.4$ with initial conditions $x(0) = 1$, $y(0) = z(0) = 0$. Suitable scaling factors are $\lambda_x = \frac{1}{3}$, $\lambda_y = \frac{1}{5}$, and $\lambda_z = \frac{1}{8}$ yielding

$$\begin{aligned}\dot{\widehat{x}} &= -\frac{5}{3}\alpha\widehat{y} + \alpha\widehat{x} - 3\alpha\widehat{x}^3,\\ \dot{\widehat{y}} &= -0.6\widehat{x} + 1.6\widehat{z}, \text{ and}\\ \dot{\widehat{z}} &= -0.625\widehat{y}\end{aligned}$$

with $\widehat{x}(0) = \frac{1}{3}$ and $\widehat{y}(0) = \widehat{z}(0) = 0$. The resulting analog computer program is shown in figure 10. As before, there is room for improvement with respect to the use of inverters. The actual implementation is shown in figure 11 while the resulting (beautiful) phase space plots are shown in figure 12.

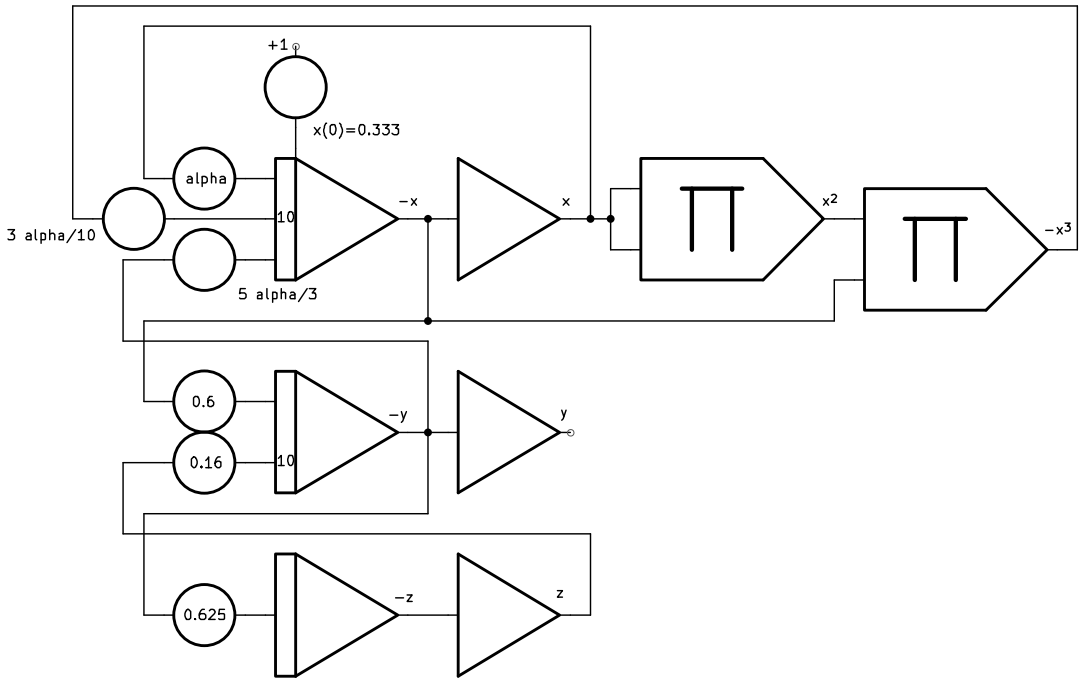


Figure 10: Analog computer program for the cubic system



Figure 11: Actual implementation of the cubic system

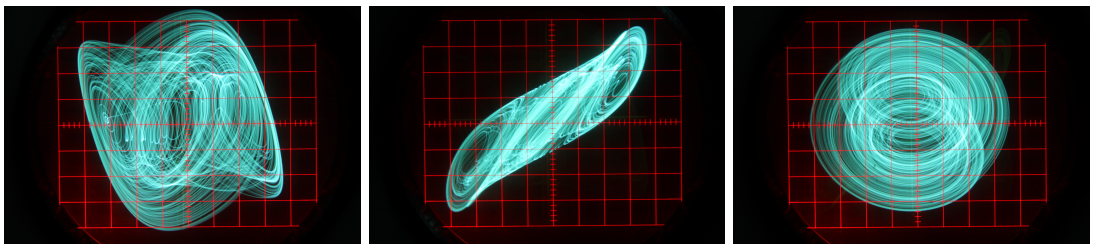


Figure 12: Double-scroll-attractor of the cubic system

References

[Ginoux et al.(2025)] Jean-Marc Ginoux, Roberto Concas, Eugenio Pugliese, Riccardo Meucci, Leon O. Chua, "From Relaxation to Chaotic Oscillations: A New Paradigm for Memristor Circuits", in *IEEE Transactions on Circuits and Systems*, I, Regular Papers, Vol. 72, No. 11, November 2025, pp. 7160–7169

[Ulmann(2016)] Bernd Ulmann, *The* Chua*-oscillator*, Analog Computer Applications, `https://analogparadigm.com/downloads/alpaca_3.pdf`

[Ulmann(2023)] Bernd Ulmann, *Analog and hybrid computer programming*, DeGruyter, 2nd edition, 2023

[Wang et al.(2012)] Xiong Wang, Guanrong Chen, "A chaotic system with only ony stable equilibrium", in *Communications in Nonlinear Science and Numerical Simulation*, Volume 17, Issue 3, March 2012, pp. 1264–1272