

The FITZHUGH-Nagumo model

Analog computers played a large role in the early days of modelling the behavior of neurons.¹ The FITZHUGH-NAGUMO model was inspired by the rather complex HODGKIN-HUXLEY model² and is described by

$$\begin{aligned}\dot{V} &= V - \frac{V^3}{3} - W + 2I_{\text{ext}} \text{ and} \\ \dot{W} &= \tau^{-1}(V + a - bW)\end{aligned}\quad (1)$$

with $a = 0.7$, $b = 0.8$, and $\tau = 12.5$.³ The variable V denotes the membrane potential while W describes a recovery variable. I_{ext} is a stimulus current applied externally to the cell.⁴ This value is the single input to the neuron and controls its overall behavior.

Scaling this system is straightforward, starting with the scaling factor $\lambda_v = \frac{1}{2}$, yielding

$$\begin{aligned}\hat{V} &= \lambda_V \left(2\hat{V} - \frac{8}{3}\hat{V}^3 - W + 2I_{\text{ext}} \right) \\ &= \hat{V} - \frac{4}{3}\hat{V}^3 - \frac{1}{2}W + I_{\text{ext}}.\end{aligned}\quad (2)$$

Of course, λ_V must be compensated for in equation 1 yielding

$$\dot{W} = 2\tau^{-1}\hat{V} + \tau^{-1}a - \tau^{-1}bW$$

in turn.

¹See [IZHIKEVICH et al. 2026]. In fact, the stories told by FITZHUGH about the temperamental analog computer beasts of his time are quite funny.

²See [RAMAN et al. 2021] (this book has quickly become one of the author's favorite books).

³See [IZHIKEVICH et al. 2026] and [IZHIKEVICH 2007, pp. 106 ff.].

⁴The factor 2 in front of this term has been chosen for convenience and will cancel out later.

A suitable scaling factor for W is $\lambda_W = \frac{1}{3}$ yielding

$$\begin{aligned}\widehat{\dot{W}} &= \lambda_W \left(2\tau^{-1}\widehat{V} + \tau^{-1}a - 3\tau^{-1}b\widehat{W} \right) \\ &= \frac{2}{3}\tau^{-1}\widehat{V} + \frac{1}{3}\tau^{-1}a - \tau^{-1}b\widehat{W}.\end{aligned}$$

This, too, requires a change to equation 2 to compensate for λ_W :

$$\widehat{\dot{V}} = \widehat{V} - \frac{4}{3}\widehat{V}^3 - \frac{3}{2}\widehat{W} + I_{\text{ext}}. \quad (3)$$

Introducing scaled parameters $a^* = \frac{1}{3}\tau^{-1}a = 0.0186$, $b^* = \tau^{-1}b = 0.064$, and $c^* = \frac{2}{3}\tau^{-1} = 0.0533$ finally results in the final second scaled equation

$$\widehat{\dot{W}} = c^*\widehat{V} + a^* - b^*\widehat{W}. \quad (4)$$

Equations (3) and (4) can be implemented directly on an analog computer such as THE ANALOG THING.⁵ Using an additional integrator, the stimulus current I_{ext} can automatically be varied as a linear ramp from 0 to 1 showing the overall behavior of the system. Figure 1 shows the resulting analog computer program. Its actual implementation on THE ANALOG THING can be seen in figure 2.

The result of one run with $0 \leq I_{\text{ext}} \leq 1$ is shown in figure 3. For values of I_{ext} smaller than a certain threshold no spiking occurs. Spiking starts in the third division of the screen shot and ends when I_{ext} exceeds a second much higher threshold.

Happy analog computing!

⁵See <https://the-analog-thing.org>, accessed 30.07.2026.

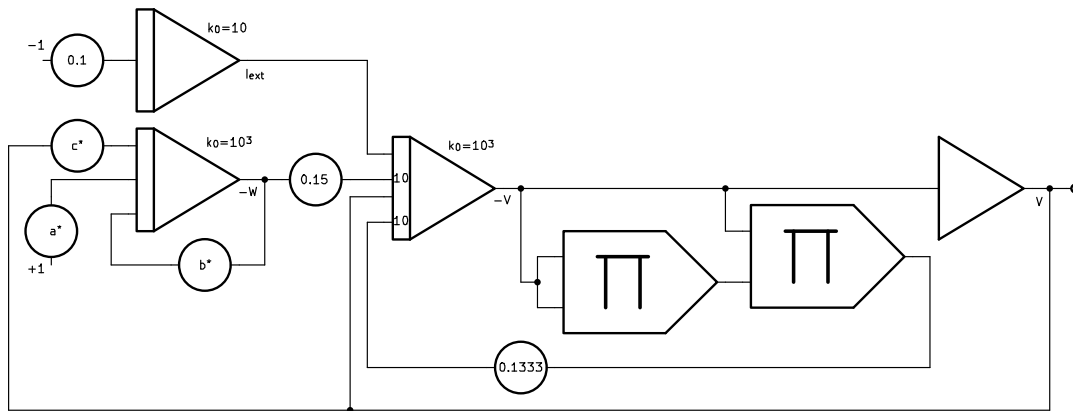


Figure 1: Analog computer program for the FitzHugh-Nagumo model

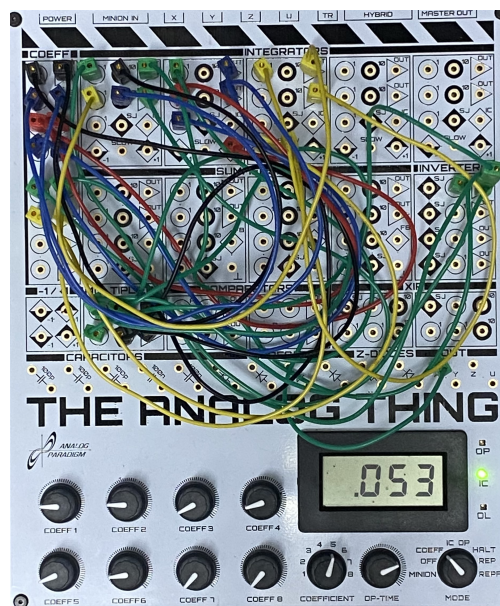


Figure 2: Implementation of the FitzHugh-Nagumo model on THE ANALOG THING

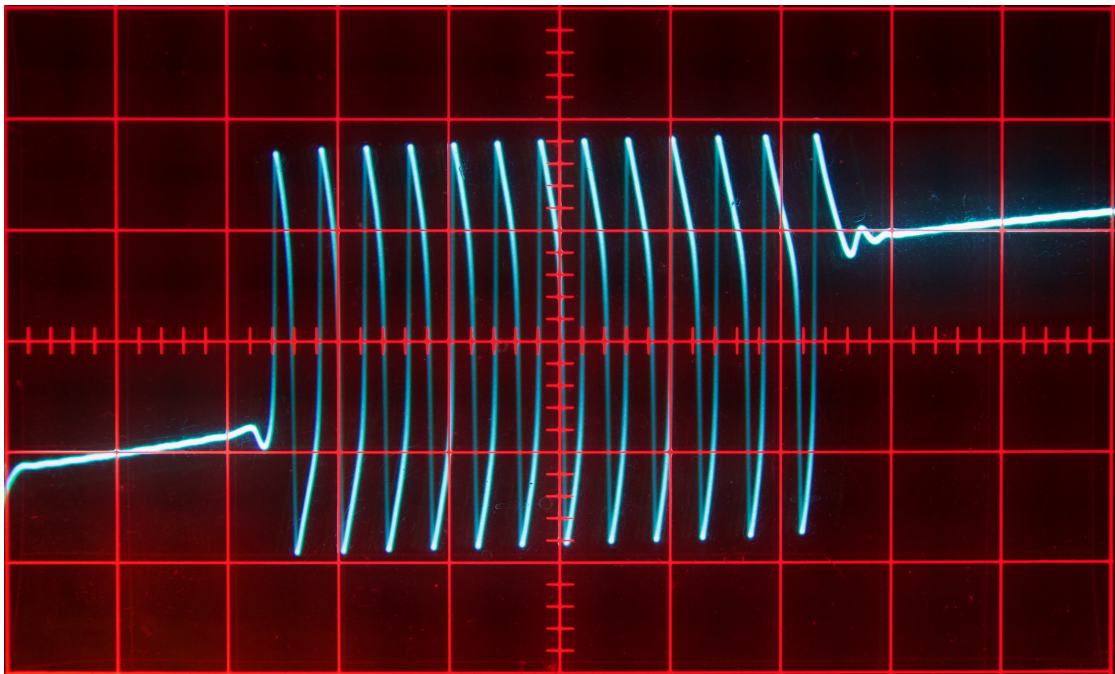


Figure 3: Typical result obtained by the FitzHugh-Nagumo model, 5 V/div vertical, 100 ms/div horizontal, $0 \leq I_{\text{ext}} \leq 1$



References

- [IZHIKEVICH 2007] EUGENE M. IZHIKEVICH, *Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting*, The MIT Press, 2007
- [IZHIKEVICH et al. 2026] EUGENE M. IZHIKEVICH, RICHARD FITZHUGH, *FitzHugh-Nagumo model*, http://www.scholarpedia.org/article/FitzHugh-Nagumo_model, accessed 12.04.2026
- [RAMAN et al. 2021] INDIRA M. RAMAN, DAVID L. FERSTER, *The Annotated HODGKIN & HUXLEY – A Reader's Guide*, Princeton University Press, 2021