

Ballistic trajectory

The following example computes the two-dimensional ballistic trajectory of a projectile fired from a cannon, taking into account the velocity dependent drag slowing the projectile down.¹ This drag causes the trajectory to deviate from a simple symmetric parabola as it will be steeper on its trailing half compared to its leading portion. The drag $\delta(v)$ is assumed to be of the general form

$$\delta(v) = rv^2 = r \left(\sqrt{\dot{x}^2 + \dot{y}^2} \right)^2$$

which is a bit oversimplified but will suffice for the following. The general equations of motion of the projectile in this two-dimensional problem are

$$\ddot{x} = -\frac{\delta(v)}{m} \cos(\varphi) \quad \text{and} \quad (1)$$

$$\ddot{y} = -g - \frac{\delta(v)}{m} \sin(\varphi) \quad (2)$$

with g representing the acceleration of gravity, v denoting the projectile's velocity, and m being its mass. Obviously, it is

$$\cos(\varphi) = \frac{\dot{x}}{v} \quad \text{and} \\ \sin(\varphi) = \frac{\dot{y}}{v}.$$

¹Cf. [KORN, 1966, pp. 2-7 ff.].

Setting the mass $m := 1$ and rearranging (1) and (2), we get the following set of differential equations:

$$\begin{aligned}\ddot{x} &= -\frac{\delta(v)}{v}\dot{x} \\ \ddot{y} &= -g - \frac{\delta(v)}{v}\dot{y}\end{aligned}$$

The computer setup resulting from these equations is shown in figures 1 and 2. The upper and lower halves of the circuit are symmetric except for the input for the gravitational acceleration to the lower left integrator yielding $\dot{y}$. The velocities $\dot{x}$ and $\dot{y}$ are fed into two multipliers yielding their respective squares which are then summed and square rooted to get v as we have $\delta(v)/v = rv$.

The parameters α_1 and α_2 are scaling parameters that are set in order to get a suitably scaled picture on an oscilloscope operated in x/y -mode. Table 1 shows the parameter set yielding the result shown in figure 3. The initial conditions satisfy

$$\begin{aligned}\dot{x}_0 &= \cos(\varphi_0) \text{ and} \\ \dot{y}_0 &= \sin(\varphi_0)\end{aligned}$$

with φ denoting the elevation of the cannon. In this example $\varphi_0 = 60^\circ$ has been chosen.

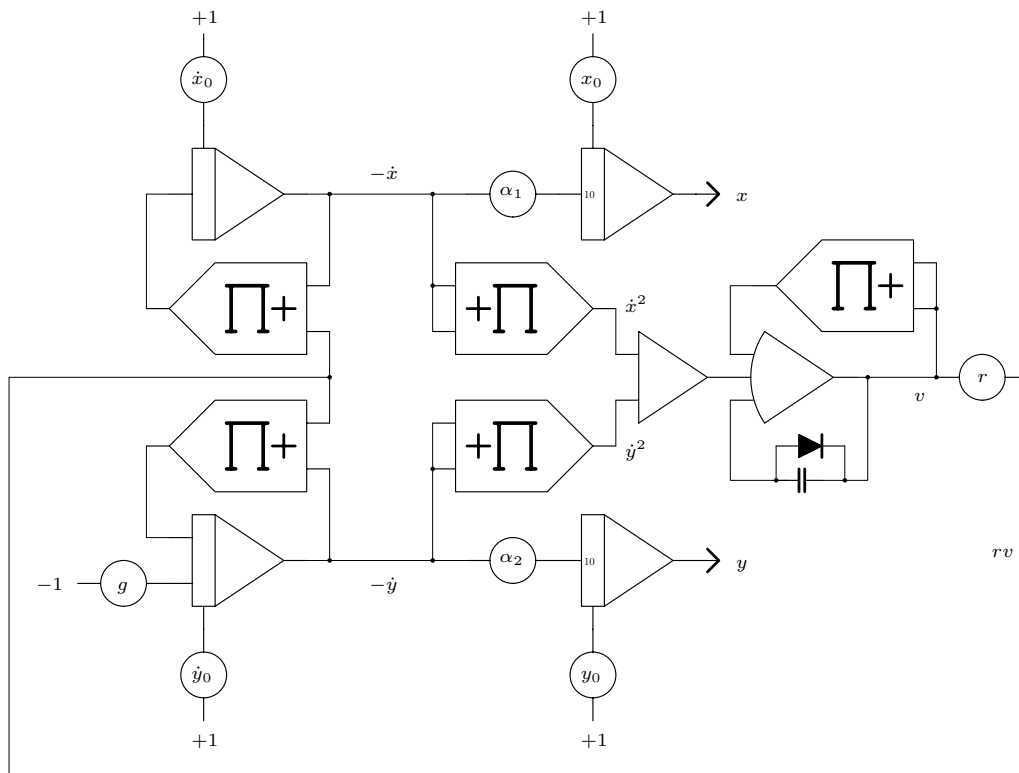


Figure 1: Computer setup for the simulation of a ballistic trajectory

Parameter	Value
$\dot{x}_0$	0.5
x_0	1
$\dot{y}_0$	0.86
y_0	1
g	0.72
r	1
α_1	0.34
α_2	0.55

Table 1: Parameter settings for the ballistic trajectory problem

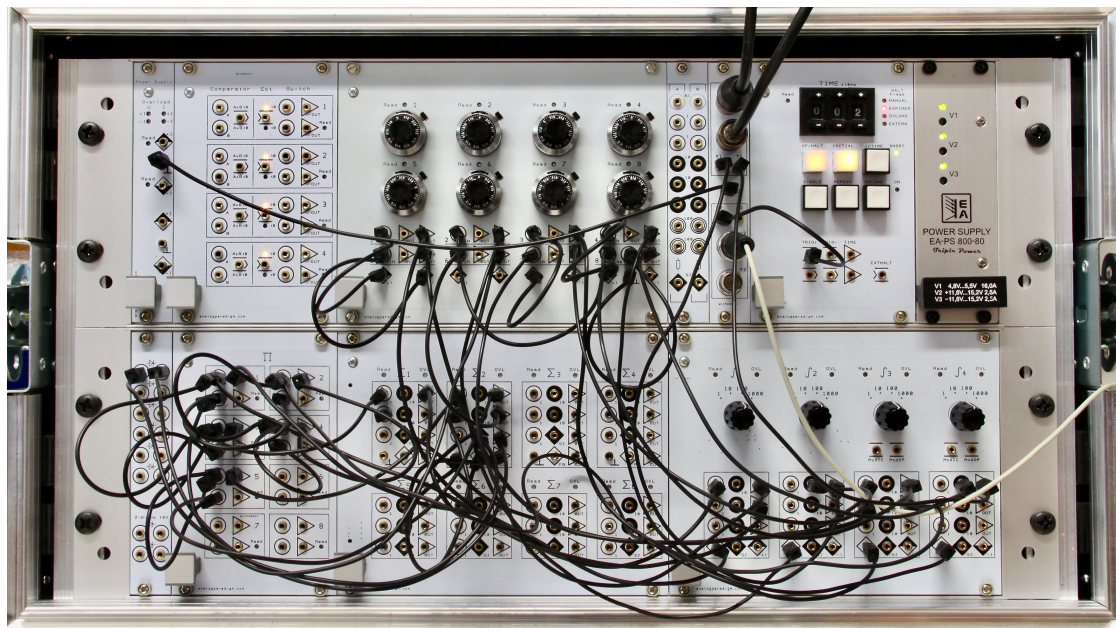


Figure 2: Setup of the ballistic trajectory problem on an Analog Paradigm Model-1 analog computer

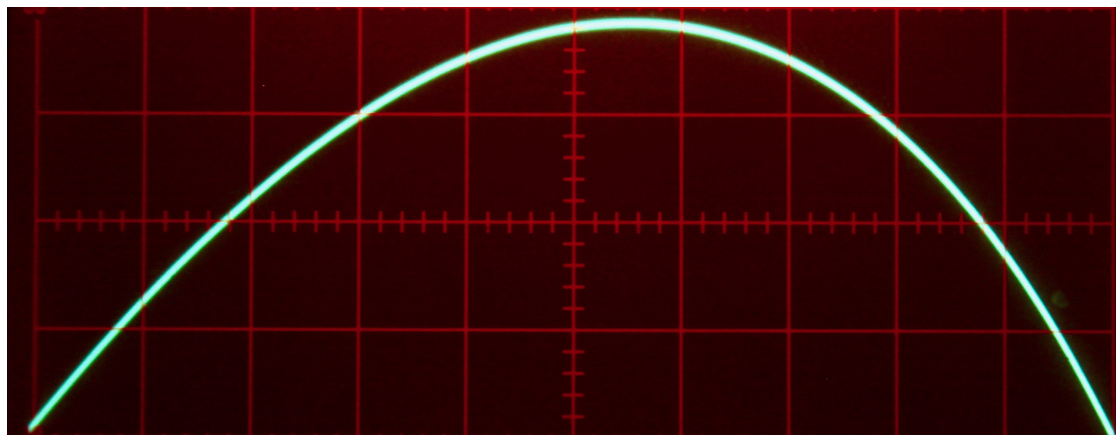


Figure 3: Ballistic trajectory



Analog Computer Applications

References

[KORN, 1966] GRANINO A. KORN, *Random-Process Simulation and Measurement*, McGraw-Hill Book Company, 1966